Drag Coefficients of NACA 0012 and NACA 4412 Airfoils from Wake-Profile Measurements

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The objective of this experiment is to determine the zero lift sectional drag coefficients of NACA 0012 and NACA 4412 airfoils from hot-wire surveys of their velocity wakes. A constant-temperature hot-wire anemometer is calibrated against an upstream Pitot-static probe in the empty test section of a wind tunnel, and a log-log power law fits the calibration with a coefficient of determination of 0.9997. Each 0.100 m chord airfoil is set near its zero lift angle, and the probe is traversed 5.56 cm vertically, one-quarter inch behind the trailing edge, at a 40 Hz fan setting. Integrating the momentum deficit across the full traverse, as prescribed, gives 0.0025 for the NACA 0012 and -0.045 for the NACA 4412. Neither value is physical. In the outer flow, where the method assumes the downstream velocity equals the upstream reference, the hot wire reads 3 to 11 percent higher, and the integral accumulates this discrepancy. Restricting the integration to the wake gives 0.015 to 0.017 for the NACA 0012 and 0.021 to 0.025 for the NACA 4412. These values have the expected sign and ordering and exceed the high-Reynolds-number data of Abbott and von Doenhoff, this is consistent with the chord Reynolds number of 2.1×10^5 and incomplete pressure recovery near the trailing edge. The likely dominant error sources are hot-wire temperature sensitivity and outer-flow nonuniformity rather than the Pitot-static calibration. The results show that a wake-derived drag coefficient is reliable only when the probe and the reference agree to well under one percent in the outer flow.

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I. Nomenclature

a	slope of the log-log hot-wire calibration
b	intercept of the log-log hot-wire calibration
c	airfoil chord
$C_{d,0}$	sectional drag coefficient at the nominal zero lift setting
D'	drag force per unit span
f	normalized momentum-deficit integrand
$F_{p,x}$	streamwise resultant pressure force on the control surface per unit span
h	measured vertical traverse height
h_1	height of the upstream control surface CS1
I	normalized wake momentum integral
L	length of the traverse lying in the outer flow outside the wake
n	number of measurement positions in a traverse
N	cumulative stepper-motor position
p	static pressure
P	atmospheric absolute pressure
q	dynamic pressure after the acquisition-system bias correction
R	specific gas constant of dry air
R^2	coefficient of determination of the calibration fit
Re_c	Reynolds number based on chord
T	absolute air temperature
u	downstream streamwise velocity
u_e	reference velocity at the edge of the wake used in the wake-bounded integral
U	Pitot-static air velocity during calibration
U_∞	mean upstream reference velocity
V	hot-wire output voltage
y	transverse position through the wake
y_1, y_2	lower and upper limits of the full-traverse integral
y_a, y_b	lower and upper limits of the wake-bounded integral
α	airfoil geometric angle of attack
$\Delta C_{d,0}$	change in the drag coefficient caused by a velocity bias in the outer flow
ε	fraction by which the hot-wire velocity exceeds the reference outside the wake
μ	dynamic viscosity
ρ	air density
σ	standard uncertainty in the indicated quantity
Subscripts	
bias	measured instrument zero offset
corrected	value after removal of the zero offset
export	value in the acquisition-system output
fit	value predicted by the calibration fit
sensor	value indicated by the pressure transducer before correction
i	index of a measurement position
∞	upstream reference condition

II. Introduction

The drag on a two-dimensional airfoil can be measured either on the model or in the flow behind it. Integrating surface pressures, as in the previous experiment, captures pressure drag but excludes skin friction, because shear stress acts tangentially to the surface and is not sensed by pressure taps. A wake survey captures both. The pressure and viscous forces on the airfoil remove streamwise momentum from the flow, producing a velocity deficit behind the trailing edge. Applying conservation of mass and momentum to a control volume enclosing the airfoil relates this deficit directly to the drag force [1, 2].

The deficit is measured with a constant-temperature hot-wire anemometer. A feedback circuit holds a heated wire at a fixed temperature, and the voltage required to do so increases as convective heat transfer to the passing air increases.

Because the output depends on heat transfer rather than directly on velocity, the probe must be calibrated against a known reference [3]. The heat transfer also depends on the temperature difference between the wire and the air, so a change in ambient temperature produces the same output change as a change in velocity.

The analysis assumes steady, incompressible flow, which is appropriate at a Mach number near 0.1, along with a uniform upstream velocity and negligible viscous stresses on the outer control surfaces. Viscous effects are retained inside the boundary layers and wake, where they produce the deficit. The standard wake equation also assumes that the net pressure force on the control surface is negligible [2]. This assumption is significant here because the probe is only a quarter inch behind the trailing edge, where the static pressure may not have recovered fully to its free stream value.

The objective is to measure the velocity wakes of a symmetric NACA 0012 and a cambered NACA 4412, determine their zero lift sectional drag coefficients, and compare the results with the previous surface-pressure experiment and published airfoil data [4, 5]. Both models have a 0.100 m chord and are tested at a 40 Hz fan setting. The accuracy of the resulting drag coefficient depends on whether the assumptions of the wake-momentum method are satisfied under the experimental conditions.

III. Experimental Procedure

A. Equipment and Test Conditions

The experiment is conducted in the low-speed, closed-test-section wind tunnel used in the previous experiments. An upstream Pitot-static probe connected to a pressure transducer provides the reference velocity, and a constant-temperature hot-wire probe measures the velocity behind the airfoil. The hot wire is mounted on a vertical traverse driven by a stepper motor with 400 steps per revolution. The traverse advances one inch per 16 revolutions, giving 3.96875×10^{-6} m per step.

The ambient pressure and temperature are recorded at the start of the procedure as 96.4 kPa and 89°F. The pressure transducer has a measured zero bias of −2.3 Pa. The data-acquisition software already removes this offset before exporting the dynamic-pressure and velocity data, so the correction is documented here but is not applied again in MATLAB. The ideal-gas law gives an air density of 1.101744 kg/m^3 at these conditions.

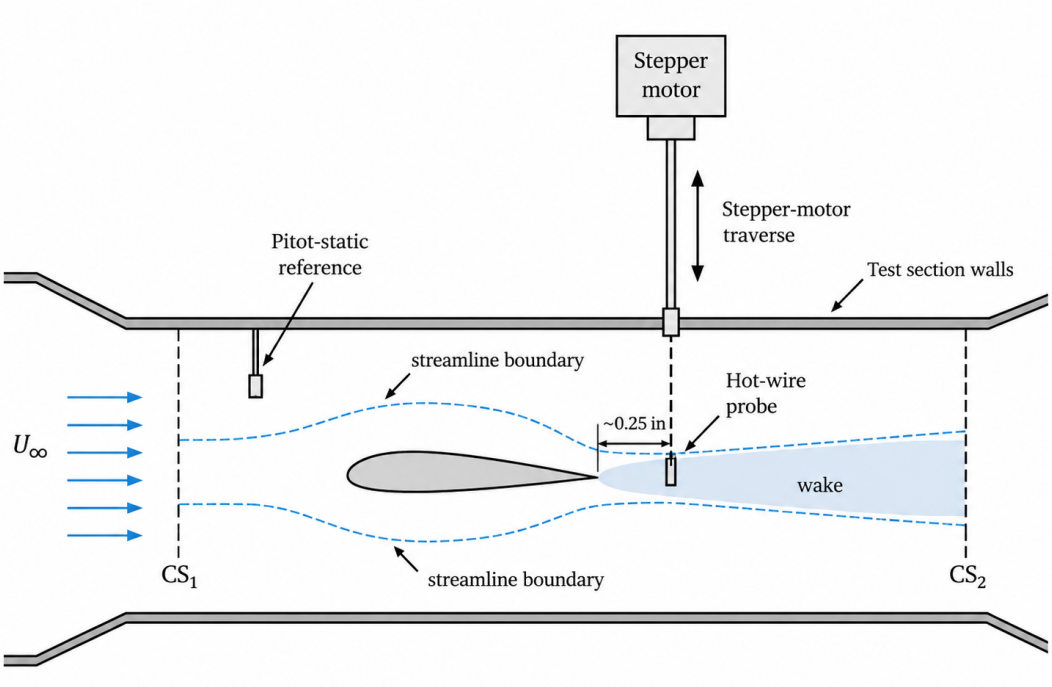


Fig. 1 Control-volume diagram and experimental arrangement for the airfoil wake measurements.

B. Hot-Wire Calibration and Wake Traverses

The hot wire is calibrated with the test section empty and the probe centered in the flow. The fan speed is increased in 5 Hz increments from 10-50 Hz, and the hot-wire voltage, dynamic pressure, and air velocity are recorded at each setting.

The nine calibration points are fit with the prescribed power law.

Each airfoil is then mounted at its zero lift angle, approximately 0° for the NACA 0012 and -4° for the NACA 4412. The probe is positioned approximately one-quarter inch behind the trailing edge and lowered near the bottom of its travel, where the traverse position is zeroed. With the fan at 40 Hz, the probe is stepped upward through the wake, and the hot-wire voltage and upstream velocity are recorded at each position.

The step size is reduced near the wake, where the velocity gradient is largest. For the NACA 0012, the probe moves in 400-step increments to 3600 steps, then 200 steps to 4800, 100 steps to 6200, 200 steps to 8400, 400 steps to 10,000, and 1000 steps to 14,000. For the NACA 4412, it moves in 400-step increments to 4400, then 200 to 5200, 100 to 7000, 200 to 8800, 400 to 10,000, and 1000 to 14,000. The traverse moves in one direction only, and the recorded positions, rather than the nominal increments, are used in all calculations.

C. Calibration, Velocity, and Drag Reduction

The transducer zero offset is handled during data acquisition. In general, a pressure reading is corrected as

$$q_{\text{corrected}} = q_{\text{sensor}} - q_{\text{bias}}, \qquad q_{\text{bias}} = -2.3 \text{ Pa}. \tag{1}$$

The exported data already contains $q_{\text{corrected}}$, so nothing further is subtracted. The upstream velocity in each wake file is also used as recorded. As instructed by the teaching assistant, its mean over each traverse is taken as the upstream velocity U_∞ at control surface CS1.

The course calibration is a power law rather than the full form of King's law:

$$\ln U = a \ln V + b, \qquad U = e^b V^a. \tag{2}$$

A least-squares fit to the logarithms of the calibration data gives a and b, and the resulting relation converts each downstream voltage into a velocity $u(y)$. The traverse position and normalized height are

$$y_i = (N_i - N_1)\frac{0.0254}{400(16)}, \qquad \frac{y_i}{h} = \frac{y_i}{y_{\text{last}}}. \tag{3}$$

The drag per unit span comes from the control-volume analysis derived in Appendix B:

$$D' = \rho \int_{y_1}^{y_2} u(y)\,[U_\infty - u(y)]\; dy. \tag{4}$$

Dividing by the dynamic pressure and chord, $C_{d,0} = D'/(\frac{1}{2}\rho U_\infty^2 c)$, gives the drag coefficient

$$C_{d,0} = \frac{2}{c}\int_{y_1}^{y_2} \frac{u(y)}{U_\infty}\left(1 - \frac{u(y)}{U_\infty}\right) dy. \tag{5}$$

Because the measurement spacing is nonuniform, the integral is evaluated with the composite trapezoidal rule using the actual position of each point:

$$I \approx \sum_{i=1}^{n-1} \frac{f_i + f_{i+1}}{2}(y_{i+1} - y_i), \qquad f_i = \frac{u_i}{U_\infty}\left(1 - \frac{u_i}{U_\infty}\right), \qquad C_{d,0} = \frac{2I}{c}. \tag{6}$$

All measured points are retained. Where the downstream velocity exceeds U_∞, the integrand is negative, and these contributions are included in the sum.

IV. Results and Discussion

A. Hot-Wire Calibration

Figures 2 and 3 show the calibration and its log-log form. The least-squares fit gives

$$U = e^{-11.900193}\, V^{12.334578}, \tag{7}$$

with a coefficient of determination of 0.999709 and a root-mean-square velocity residual of 0.2622 m/s. Although the fit is close, the residuals in Appendix A, Table A-2, are systematic. They are positive at both ends of the range and negative in the middle, suggesting that a two-parameter power law doesn't completely capture the curvature of the anemometer

response. At the survey speed the residual is 0.033 m/s, so the functional form of the fit contributes only a little error to the wake measurements.

The magnitude of the exponent is vastly more consequential. Because velocity varies approximately as the twelfth power of voltage, a one percent change in voltage produces a twelve percent change in velocity. The exported voltage resolution of 1 mV corresponds to a velocity increment of 0.35 percent, or 0.12 m/s, and any drift in the anemometer output is amplified by the same factor. This is compounded by the fact that the air temperature also rises from 28.8 to 31.8°C during calibration as the fan speed increases, so temperature and velocity are not varied independently. The calibration point closest to the survey speed is taken at 31.3°C, while the NACA 0012 and NACA 4412 surveys average 30.0 and 29.2°C. A high coefficient of determination therefore does not demonstrate that the probe can resolve differences of one percent or less between the outer flow and the upstream reference, which is the agreement the drag integral requires (Section IV.G).

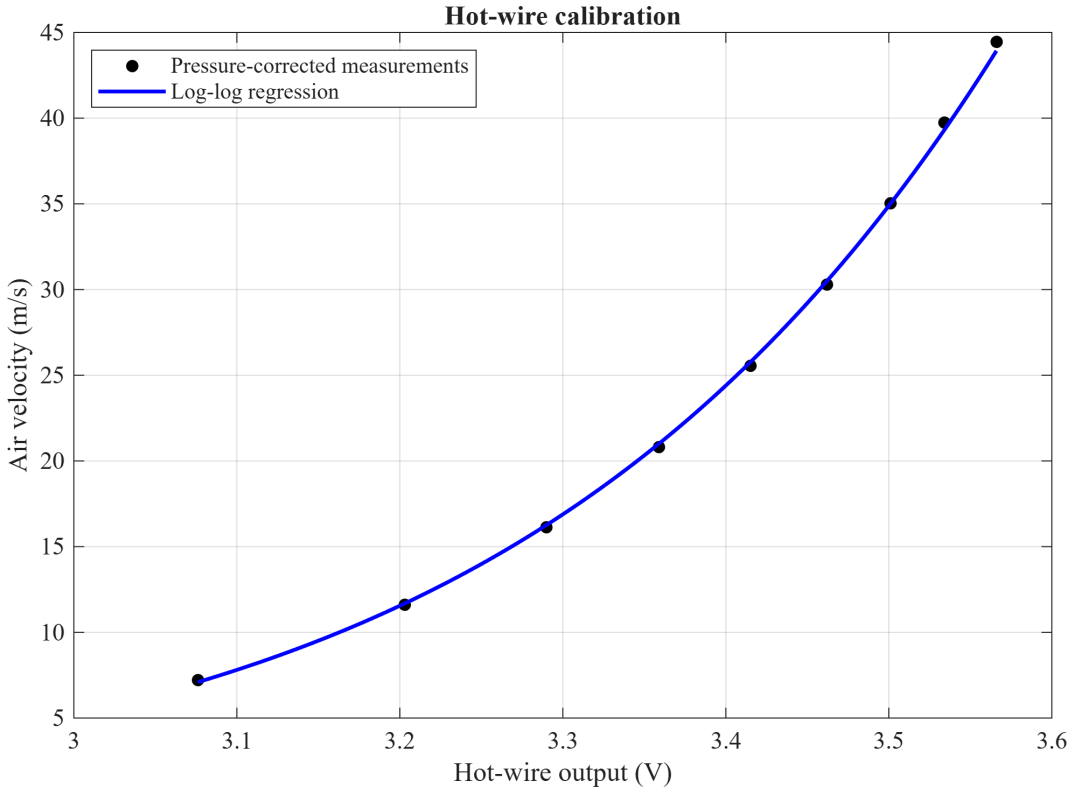


Fig. 2 Hot-wire calibration against the upstream Pitot-static velocity in the empty test section. The steep power-law response amplifies small voltage changes into large velocity changes.

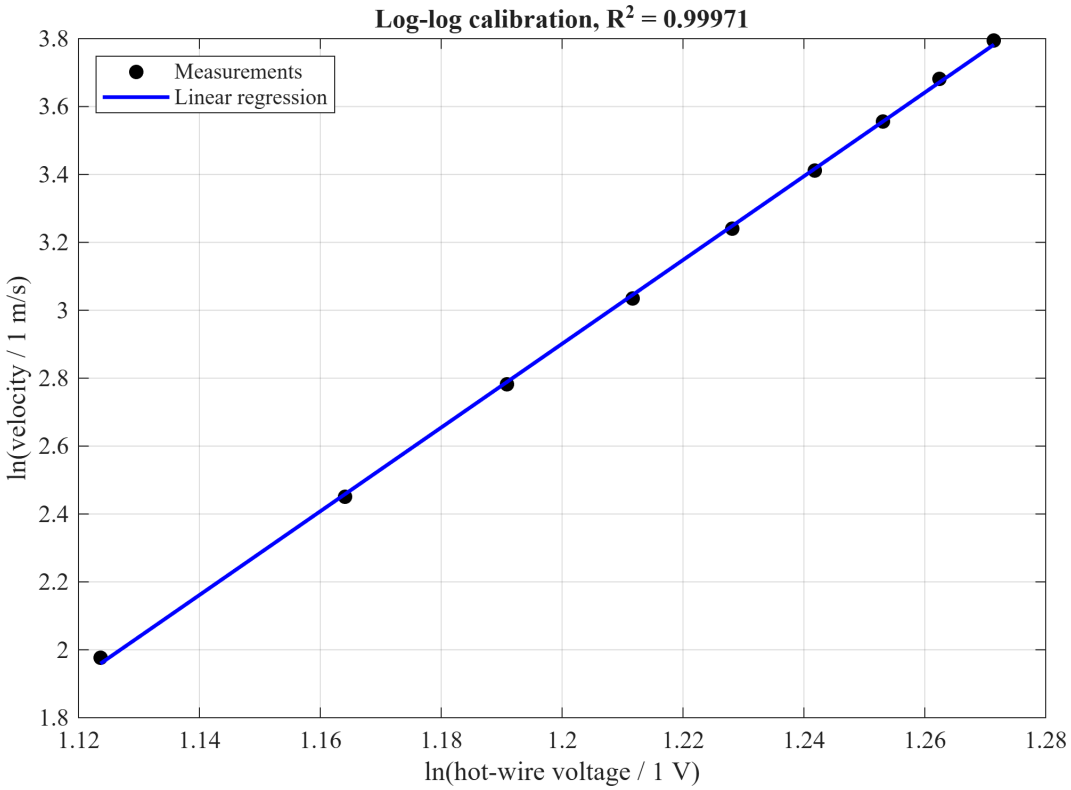


Fig. 3 Log-log calibration and linear regression, with R^2 = 0.999709.

B. NACA 0012 Velocity Profile

The NACA 0012 survey contains 48 positions over 5.557 cm, with a mean upstream velocity of 35.305 m/s. The dimensional and normalized profiles appear in Figs. 4 and 5, and the values are listed in Appendix A, Table A-3. The upstream Pitot velocity is nearly constant during the traverse, with a standard deviation of just 0.028 m/s.

The wake is well defined between 1.59 and 2.70 cm. The velocity reaches a minimum of 27.88 m/s, or 0.790 of U_∞, at 2.14 cm, and the profile is nearly symmetric about this point, this is as expected for a symmetric section at zero lift. Both edges of the wake level off at a normalized velocity of 0.988, and the outer flow above the wake increases gradually to 1.013 at the top of the traverse. An edge velocity slightly below the upstream value that recovers with distance from the wake is seemingly consistent with a static pressure above the free-stream value near the trailing edge, where pressure recovery is incomplete.

Below the wake, the profile changes notably abruptly. In the section between 1.43 and 1.59 cm, the hot-wire velocity increases from 0.988 to 1.038 of U_∞ while the upstream Pitot reading is unchanged, and every point below this step reads 3 to 5 percent above the reference. Ten of the fourteen points that exceed U_∞ lie in this region, and the remaining four lie near the top of the traverse. Because the step lies outside the wake and is not accompanied by a change in tunnel speed, it is not part of the momentum deficit produced by the airfoil.

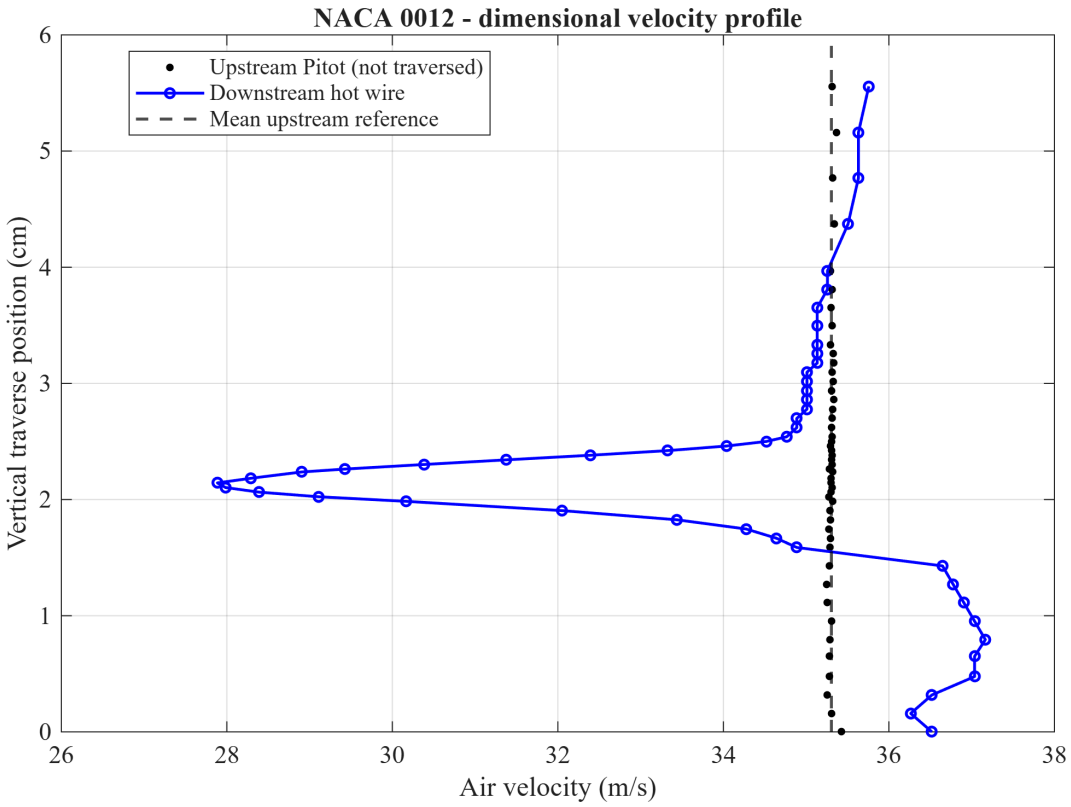


Fig. 4 Measured upstream and downstream velocities for the NACA 0012. The wake lies between 1.59 and 2.70 cm, and the region below 1.43 cm reads faster than the upstream reference.

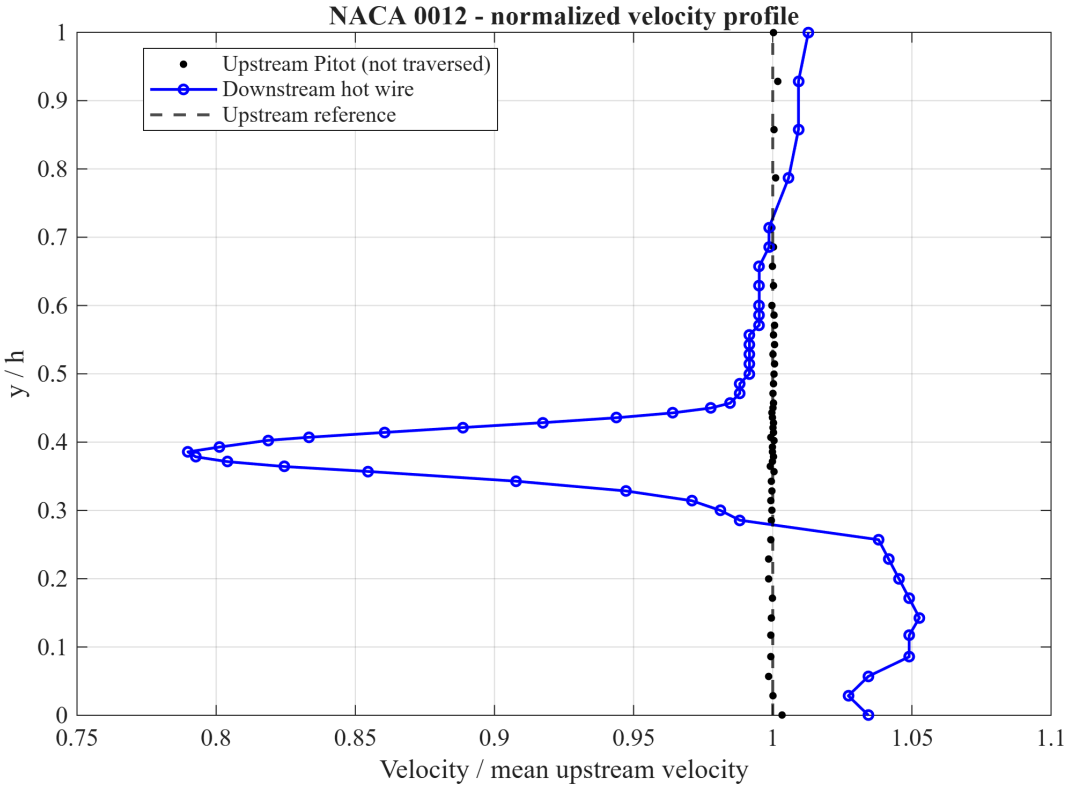


Fig. 5 Normalized upstream and downstream velocity profiles for the NACA 0012.

C. NACA 4412 Velocity Profile

The NACA 4412 survey contains 50 positions over 5.556 cm, with a mean upstream velocity of 34.902 m/s and a Pitot standard deviation of 0.023 m/s. The profiles are in Figs. 6 and 7, and the values are in Appendix A, Table A-4. The main deficit extends from roughly 1.83 to 2.86 cm and reaches a minimum of 24.67 m/s, or 0.707 of U_∞, near 2.42 cm. The deficit is deeper than for the NACA 0012, and the profile is asymmetric. Asymmetry is expected behind a cambered section, because the upper and lower boundary layers develop under different pressure gradients.

The outer flow is considerably less uniform than for the NACA 0012. Below the wake, the hot wire reads approximately 0.975 of U_∞ between 1.27 and 1.83 cm, then increases to between 1.03 and 1.05 below 0.95 cm, resembling the lower step in the NACA 0012 survey. Above the wake, the velocity reaches 1.08 of U_∞ within 0.2 cm of the wake edge and continues to increase to 1.11 at the top of the traverse. Twenty-seven of the 50 points exceed U_∞, eight below the wake and nineteen above it, so the outer flow on the two sides of the wake differs by approximately ten percent. A uniform calibration error in either instrument scales every point by the same factor and therefore could not have produced this asymmetry.

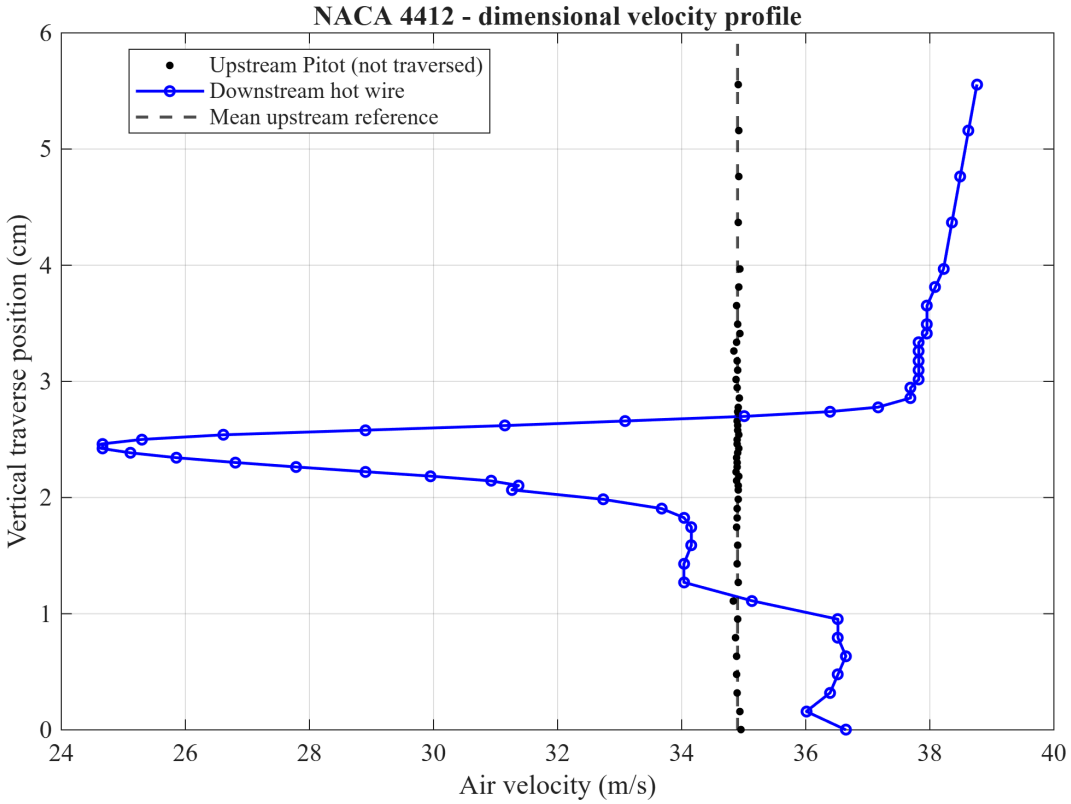


Fig. 6 Measured upstream and downstream velocities for the NACA 4412. The outer flow above the wake reads 8 to 11 percent faster than the upstream reference.

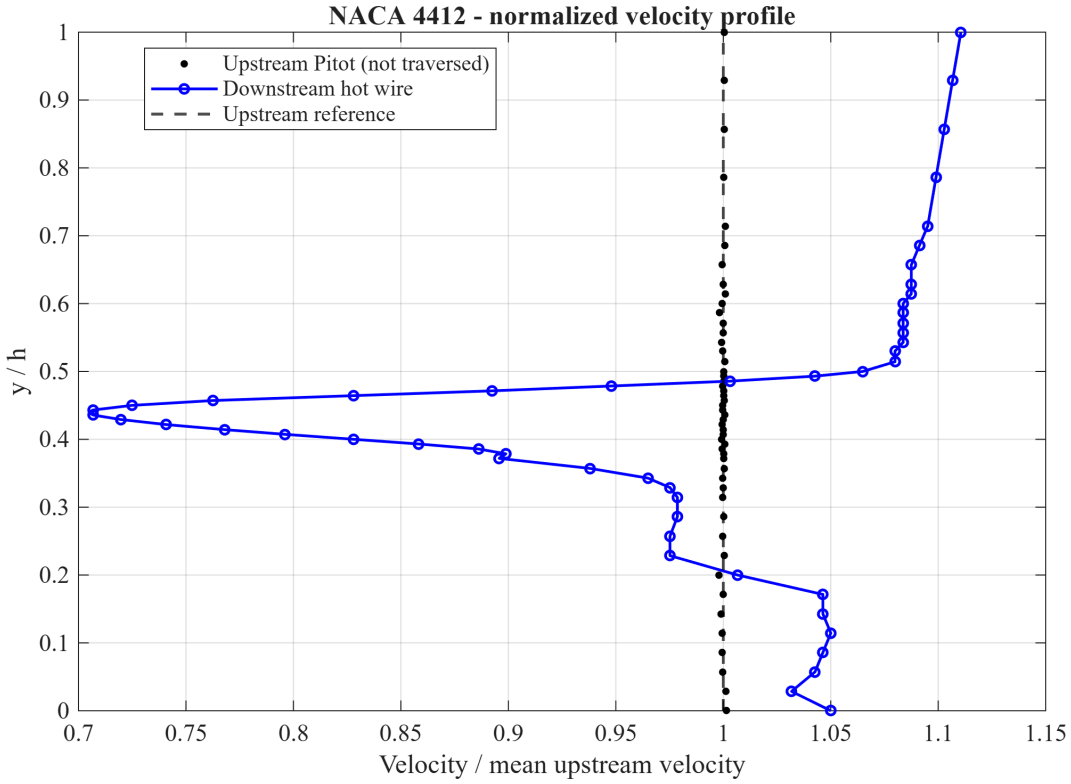


Fig. 7 Normalized upstream and downstream velocity profiles for the NACA 4412.

D. Sectional Drag Coefficients

Integrating Eq. (5) across the full traverse with the mean upstream velocity as the reference, as prescribed, gives a drag coefficient of 0.002516 for the NACA 0012 and −0.044857 for the NACA 4412 (Table 2). For the NACA 0012, a positive contribution of 0.018144 is nearly cancelled by a negative contribution of 0.015628. For the NACA 4412, the negative contribution of 0.068431 is nearly three times the positive contribution of 0.023574. Neither total is a credible section drag coefficient. A negative value is physically impossible for a stationary, unpowered airfoil, and the NACA 0012 value of 0.0025 is below the high-Reynolds-number data [4], although the drag at the present Reynolds number should be higher.

Table 1 separates each integral by region. Within the wake, both integrals are positive and of physically reasonable magnitude. The negative contributions arise entirely outside the wake, where the derivation in Appendix B assumes the integrand is zero. For the NACA 0012, the region below 1.43 cm cancels most of the wake contribution. For the NACA 4412, the outer flow above the wake contributes −0.0587, nearly three times the contribution of the wake itself.

Table 1 Contributions to $C_{d,0}$ from each region of the traverse, using the mean upstream reference.

Airfoil	Region	Contribution
NACA 0012	Lower outer flow, 0 to 1.43 cm	−0.0127
	Step, 1.43 to 1.59 cm	−0.0004
	Wake, 1.59 to 2.70 cm	0.0167
	Upper outer flow, 2.70 to 5.56 cm	−0.0010
	Total	0.0025
NACA 4412	Lower outer flow, 0 to 0.95 cm	−0.0088
	Lower plateau, 0.95 to 1.83 cm	0.0019
	Wake, 1.83 to 2.70 cm	0.0208
	Upper outer flow, 2.70 to 5.56 cm	−0.0587
	Total	−0.0449

The control volume in Appendix B places its outer boundaries in flow that has returned to U_∞. Integrating through outer flow that does not satisfy this condition adds the disagreement between the two instruments to the result rather than any momentum deficit. An estimate bounding the wake, or a wake-bounded estimate is therefore also reported, with the integration limits y_a and y_b placed at the plateaus on either side of the wake:

$$C_{d,0} \approx \frac{2}{c} \int_{y_a}^{y_b} \frac{u}{u_e} \left(1 - \frac{u}{u_e}\right) dy. \tag{8}$$

The reference u_e is taken either as the mean upstream velocity or as a local edge velocity that varies linearly between the hot-wire readings at the two wake edges. The edge reference is measured by the same instrument as u, so any uniform hot-wire bias cancels. The results in Table 2 range from 0.0146 to 0.0167 for the NACA 0012 and from 0.0208 to 0.0226 for the NACA 4412. Moving the limits by up to two measurement points along the adjacent plateaus keeps the values between 0.015 and 0.017 for the NACA 0012 and between 0.021 and 0.025 for the NACA 4412. The wake-bounded values are taken as the best available estimates, and the full-traverse values are retained as the output of the prescribed method.

Table 2 Sectional results from the full traverse and from the wake-bounded integration.

Quantity	NACA 0012	NACA 4412
U_∞, m/s	35.3051	34.9017
Re_c	208,160	205,782
I, full traverse, m	0.000125807	−0.002242861
D', full traverse, N/m	0.17277	−3.01007
$C_{d,0}$, full traverse	0.002516	−0.044857
Wake limits, cm	1.59 to 2.70	1.83 to 2.70
$C_{d,0}$, wake-bounded, u_e from U_∞	0.0167	0.0208
Wake limits, cm	1.59 to 2.70	1.83 to 2.86
$C_{d,0}$, wake-bounded, u_e from wake edges	0.0146	0.0226

E. Comparison with Expectations and the Previous Experiment

Both sections are expected to have positive drag, with the NACA 4412 at least equal to the NACA 0012. Abbott and von Doenhoff report minimum section drag coefficients of approximately 0.006 for both smooth sections at Reynolds numbers of 3 to 9 million [4]. The wake-bounded results have the expected sign and ordering but are two and a half to four times the published values. Three effects act in this direction. First, the chord Reynolds number of 2.1×10^5 is more than an order of magnitude below that of [4], so the boundary layers would in theory remain laminar over a larger fraction of the chord and are therefore susceptible to laminar separation, both of which increase profile drag. Second, the probe is approximately six percent of a chord behind the trailing edge. If the static pressure there exceeds the free-stream value, as the NACA 0012 edge velocities of 0.988 indicate, part of the measured deficit is a pressure effect rather than a momentum loss, and the simplified integral overestimates the drag. And third, the NACA 4412 may not be exactly at zero lift (Section IV.F).

The previous report gave pressure-only drag coefficients of 0.0151 for the NACA 0012 and 0.0242 for the NACA 4412, both at 0° [5]. Because the wake method includes both pressure drag and skin friction, it should exceed the pressure-only value at the same condition. For the NACA 0012, the wake-bounded result of 0.015 to 0.017 leaves little margin for skin friction, which indicates that the earlier pressure-only value is overestimated, a common outcome when a small drag force is computed from a limited number of surface taps near the trailing edge. The NACA 4412 values are not directly comparable, because the earlier result is at 0°, where the section generates lift, whereas the present test is near its zero lift angle.

F. Effect of a Nonzero Angle of Attack

A nonzero angle of attack affects the wake method in four ways. Downwash deflects the wake of a lifting section, so its center shifts vertically and a traverse of fixed extent may not capture both edges. The suction-side boundary layer thickens, producing a wider, more asymmetric deficit and higher profile drag. The wake is inclined to the tunnel axis, while the single hot wire responds to the total cooling velocity rather than only its streamwise component. And finally, the circulation alters the pressure field near the trailing edge, increasing the pressure term neglected in Eq. (4). These effects are most relevant to the NACA 4412. The course setting of −4° is close to the thin-airfoil zero lift angle, but the lift curve from the previous experiment indicates a zero lift angle nearer −6.4° [5]. Although its important to note that in the previous experiemnt, one pressure transducer was faulty on the 4412. Nevertheless, in that case, the section carries a lift coefficient of approximately 0.26 during the survey. A lumped-vortex estimate places the resulting change in streamwise velocity at the survey plane at approximately one percent, which may contribute to the asymmetry of the NACA 4412 outer flow but cannot account for a ten percent difference.

G. Error Sources and Sensitivity

The full-traverse result is highly sensitive to any disagreement between the hot wire and the upstream reference outside the wake. If the hot wire reads a small fraction ε above the reference there, the integrand in Eq. (5) becomes approximately the negative of ε. Over an outer-flow length L, the drag coefficient shifts by

$$\Delta C_{d,0} \approx -\frac{2}{c}\int_L \varepsilon \, dy \approx -\frac{2\varepsilon L}{c}. \tag{9}$$

With a 0.100 m chord and approximately 4 cm of outer flow in each survey, a one percent bias shifts the drag coefficient by approximately 0.009. Applying this bias to the measured data lowers the NACA 0012 result from 0.0025 to −0.0086

and the NACA 4412 result from −0.045 to −0.057. In addition to this, the NACA 4412 integral would vanish if the reference velocity were 36.269 m/s, 3.9 percent above the measured mean. The prescribed method therefore requires agreement between the probe and the reference to well under one percent, which is comparable to the voltage resolution of the exported data.

The upstream Pitot-static probe is an unlikely source of the discrepancy. Its readings vary by less than 0.1 percent during each traverse, and the −2.3 Pa bias corresponds to 0.17 percent of velocity at the survey speed. More importantly, the hot wire is calibrated against the same Pitot-static system, so any scale error in the Pitot reading is common to u and U_∞ and cancels in their ratio.

The hot wire is the more probable and logical source. A constant-temperature anemometer reads high when the ambient air is cooler than during calibration, because the wire overheat ratio increases [3]. Both surveys are performed 1.3 to 2.1 K below the temperature of the calibration point nearest the survey speed. A standard temperature correction, with an assumed wire temperature between 200 and 300°C, changes the full-traverse coefficients to 0.030 to 0.045 for the NACA 0012 and 0.007 to 0.033 for the NACA 4412. Because the wire temperature is not recorded, this correction is not adopted. However, it shows that a temperature difference of one to two kelvins changes the result by more than the expected drag coefficient. Although temperature does not account for every feature. In the upper outer flow of the NACA 4412, the air temperature rises slightly, which would lower the indicated velocity, yet the measured velocity increases monotonically.

The abrupt steps in the lower outer flow of both surveys seem to indicate an additional error source. Both occur at similar traverse heights without a change in the Pitot reading, and the NACA 4412 step coincides with a temperature rise of approximately 0.5 K. Because each traverse proceeds in one direction only, position and elapsed time are confounded, and the data cannot distinguish a spatial nonuniformity in the test section from drift in the anemometer output. The density uncertainty of 0.0038 kg/m^3, which uses the instrument specifications assumed from Lab 1, affects D' but cancels out of $C_{d,0}$. No total uncertainty is assigned to $C_{d,0}$, because the hot-wire temperature response, traverse accuracy, and static pressure at the survey plane are not specified.

Several changes could reduce these errors. The survey should be performed farther downstream, where the static pressure has recovered, and extended until the velocity reaches a plateau on both sides of the wake. A second traverse in the opposite direction, or repeated readings at a fixed position, would separate spatial structure from drift. Recording the hot-wire voltage in the empty tunnel immediately before and after each survey, calibrating at the survey temperature, and recording the overheat setting of the wire would allow a temperature correction to be applied.

V. Conclusion

Hot-wire surveys of the wakes of a NACA 0012 and a NACA 4412 near zero lift are used to estimate their sectional drag coefficients. Both wakes are clearly resolved. The NACA 0012 wake is nearly symmetric with a minimum of 0.79 of the free-stream velocity, and the NACA 4412 wake is deeper and asymmetric, which as expected for a cambered airfoil. Integrating across the full traverse, as prescribed, gives 0.0025 for the NACA 0012 and −0.045 for the NACA 4412. These values are not physical drag coefficients. They result from outer flow that reads several percent above the upstream reference in the region where the method assumes the two are equal, and the integral accumulates this discrepancy over several centimeters. Restricting the integration to the wake gives 0.015 to 0.017 for the NACA 0012 and 0.021 to 0.025 for the NACA 4412, which have the expected sign and ordering.

The steady, incompressible flow assumptions are valid at a Mach number near 0.1, and the viscous wake is treated as nonuniform. The assumptions that are not satisfied are those at the control-volume boundaries, specifically that the outer flow has returned to the upstream velocity and that the static pressure has recovered at the survey plane. The wake-bounded results exceed the high-Reynolds-number data of Abbott and von Doenhoff, consistent with the lower Reynolds number and with incomplete pressure recovery a quarter inch behind the trailing edge. This evidence points to hot-wire temperature sensitivity and outer-flow non uniformity, rather than the Pitot-static calibration, as the dominant sources of error. Surveying farther downstream, capturing both edges of the wake, reversing the traverse, and calibrating at the survey temperature could address them.

Wake surveys are a standard method for measuring profile drag in two-dimensional wind-tunnel and flight testing, because they isolate the momentum removed by the section from support and wall forces. This experiment shows that the accuracy of the method depends less on resolving the wake deficit than on the agreement between the probe and the reference in the outer flow, where a one percent disagreement changes the drag coefficient by more than half its expected value. Any measurement that infers a small aerodynamic force from the difference between two instruments must therefore control their relative calibration as carefully as it measures the effect itself.

Appendix A: Tabulated Data

The following tables list the processed values used in the calibration and velocity-profile plots. They are reduced measurements rather than a copy of the raw acquisition files.

Table A-1 Ambient conditions, density, and provisional instrument uncertainty.

Quantity	Result
Absolute pressure	96,400 Pa
Temperature	304.817 K (31.667°C)
Pressure uncertainty	100 Pa (provisional, from Lab 1)
Temperature uncertainty	1 K (provisional, from Lab 1)
Density	1.101744 kg/m^3
Density uncertainty	0.003791 kg/m^3
Pressure zero bias (already corrected in export)	−2.3 Pa

Table A-2 Hot-wire calibration data after acquisition-system pressure-bias correction.

V, V	q, Pa	U_{measured}, m/s	U_{fit}, m/s	Residual, m/s
3.076	28.97	7.213	7.094	+0.119
3.203	74.80	11.598	11.685	-0.087
3.290	144.61	16.142	16.263	-0.121
3.359	239.83	20.803	21.008	-0.205
3.415	361.08	25.539	25.761	-0.222
3.462	507.49	30.293	30.492	-0.199
3.501	678.17	35.043	35.010	+0.033
3.534	870.87	39.726	39.305	+0.421
3.566	1089.18	44.450	43.927	+0.523

Table A-3 NACA 0012 normalized velocity profile.

Steps	y, cm	y/h	$U_{\text{upstream}}/U_\infty$	u/U_∞
0	0.0000	0.0000	1.0034	1.0344
400	0.1588	0.0286	1.0001	1.0272
800	0.3175	0.0571	0.9986	1.0344
1200	0.4763	0.0857	0.9992	1.0490
1639	0.6505	0.1171	0.9993	1.0490
2000	0.7937	0.1428	0.9994	1.0527
2400	0.9525	0.1714	1.0000	1.0490
2801	1.1116	0.2000	0.9986	1.0453
3200	1.2700	0.2285	0.9985	1.0417
3600	1.4287	0.2571	0.9994	1.0380
4000	1.5875	0.2857	0.9994	0.9882
4200	1.6669	0.3000	0.9996	0.9812
4400	1.7462	0.3142	0.9992	0.9709
4600	1.8256	0.3285	0.9996	0.9472
4800	1.9050	0.3428	0.9995	0.9077
5000	1.9844	0.3571	1.0005	0.8545
5100	2.0241	0.3642	0.9992	0.8245
5200	2.0637	0.3714	0.9998	0.8041
5300	2.1034	0.3785	1.0002	0.7927
5400	2.1431	0.3857	0.9999	0.7898
5500	2.1828	0.3928	0.9999	0.8012
5637	2.2372	0.4026	1.0005	0.8186
5700	2.2622	0.4071	0.9993	0.8334

Table A3 continued

Steps	y, cm	y/h	$U_{\text{upstream}}/U_\infty$	u/U_∞
5800	2.3019	0.4142	1.0003	0.8606
5900	2.3416	0.4214	1.0001	0.8886
6000	2.3813	0.4285	1.0002	0.9175
6100	2.4209	0.4357	1.0000	0.9438
6200	2.4606	0.4428	0.9997	0.9641
6300	2.5003	0.4499	1.0000	0.9778
6400	2.5400	0.4571	1.0002	0.9847
6600	2.6194	0.4714	1.0000	0.9882
6800	2.6987	0.4856	1.0002	0.9882
7000	2.7781	0.4999	1.0004	0.9916
7204	2.8591	0.5145	1.0007	0.9916
7400	2.9369	0.5285	1.0001	0.9916
7600	3.0162	0.5428	1.0006	0.9916
7802	3.0964	0.5572	1.0002	0.9916
8000	3.1750	0.5713	1.0007	0.9951
8202	3.2552	0.5858	1.0005	0.9951
8400	3.3337	0.5999	0.9997	0.9951
8808	3.4957	0.6291	1.0003	0.9951
9200	3.6512	0.6570	0.9998	0.9951
9600	3.8100	0.6856	1.0003	0.9987
10000	3.9688	0.7142	0.9996	0.9987
11017	4.3724	0.7868	1.0010	1.0057
12012	4.7673	0.8579	1.0004	1.0093
13000	5.1594	0.9284	1.0018	1.0093
14002	5.5570	1.0000	1.0002	1.0128

Table A-4 NACA 4412 normalized velocity profile.

Steps	y, cm	y/h	$U_{\text{upstream}}/U_\infty$	u/U_∞
0	0.0000	0.0000	1.0015	1.0500
400	0.1588	0.0286	1.0011	1.0317
802	0.3183	0.0573	0.9998	1.0427
1203	0.4774	0.0859	0.9994	1.0463
1601	0.6354	0.1144	0.9995	1.0500
2000	0.7937	0.1429	0.9989	1.0463
2400	0.9525	0.1714	1.0001	1.0463
2800	1.1112	0.2000	0.9980	1.0066
3200	1.2700	0.2286	1.0004	0.9752
3600	1.4287	0.2571	0.9997	0.9752
4003	1.5887	0.2859	1.0002	0.9786
4401	1.7466	0.3144	0.9996	0.9786
4604	1.8272	0.3289	0.9999	0.9752
4800	1.9050	0.3429	0.9998	0.9649
5000	1.9844	0.3571	1.0004	0.9380
5207	2.0665	0.3719	1.0003	0.8957
5301	2.1038	0.3786	1.0003	0.8989
5400	2.1431	0.3857	0.9995	0.8862
5504	2.1844	0.3931	1.0006	0.8582
5600	2.2225	0.4000	0.9993	0.8281
5703	2.2634	0.4074	0.9999	0.7961
5800	2.3019	0.4143	0.9999	0.7680
5904	2.3432	0.4217	0.9995	0.7408
6009	2.3848	0.4292	1.0000	0.7197
6101	2.4213	0.4358	1.0006	0.7067

Table A4 continued

Steps	y, cm	y/h	$U_{\text{upstream}}/U_\infty$	u/U_∞
6204	2.4622	0.4431	0.9997	0.7067
6300	2.5003	0.4500	0.9998	0.7249
6400	2.5400	0.4571	1.0005	0.7625
6501	2.5801	0.4644	1.0001	0.8281
6600	2.6194	0.4714	1.0001	0.8925
6700	2.6591	0.4786	0.9997	0.9480
6800	2.6987	0.4857	1.0002	1.0031
6903	2.7396	0.4931	1.0001	1.0427
7000	2.7781	0.5000	1.0003	1.0649
7200	2.8575	0.5143	1.0008	1.0799
7420	2.9448	0.5300	0.9998	1.0799
7600	3.0162	0.5429	0.9993	1.0837
7800	3.0956	0.5571	1.0000	1.0837
8000	3.1750	0.5714	0.9998	1.0837
8218	3.2615	0.5870	0.9982	1.0837
8402	3.3345	0.6001	0.9995	1.0837
8602	3.4139	0.6144	1.0010	1.0875
8800	3.4925	0.6286	1.0000	1.0875
9200	3.6512	0.6571	0.9995	1.0875
9603	3.8112	0.6859	1.0007	1.0913
10000	3.9688	0.7143	1.0011	1.0951
11003	4.3668	0.7859	1.0003	1.0990
12000	4.7625	0.8571	1.0005	1.1028
13003	5.1606	0.9288	1.0005	1.1067
14000	5.5562	1.0000	1.0004	1.1105

Table A-5 Wake-derived drag estimates and comparison values.

Quantity	NACA 0012	NACA 4412
U_∞, m/s	35.3051	34.9017
Re_c	208,160	205,782
$C_{d,0}$, full traverse	0.002516	−0.044857
Positive contribution, full traverse	0.018144	0.023574
Negative contribution, full traverse	−0.015628	−0.068431
D', full traverse, N/m	0.17277	−3.01007
$C_{d,0}$, wake-bounded, u_e from U_∞	0.0167	0.0208
$C_{d,0}$, wake-bounded, u_e from wake edges	0.0146	0.0226
Range with limits moved along the plateaus	0.015 to 0.017	0.021 to 0.025
Lab 2 pressure-only C_d at 0° [5]	0.0151	0.0242
Abbott and von Doenhoff [4], smooth, Re of 3×10^6	≈ 0.006	≈ 0.006

Appendix B: Derivation and Sample Calculations

B.1 Continuity and Streamwise Momentum

Consider a two dimensional streamtube bounded upstream by CS1 and downstream by CS2, with the airfoil inside the control volume shown in Fig. 1. The side boundaries follow streamlines, so there is no mass flux through them. For steady, incompressible flow of unit span, the mass flow rate through CS1 equals that through CS2:

$$U_\infty h_1 = \int_{y_1}^{y_2} u(y)\, dy. \tag{10}$$

In the streamwise direction, the net external force on the fluid equals the net momentum flux out of the control volume. The airfoil exerts a force on the fluid equal and opposite to the drag, and pressure acts on the control surfaces, so

$$-D' + F_{p,x} = \rho \int_{y_1}^{y_2} u^2(y)\, dy - \rho U_\infty^2 h_1. \tag{11}$$

Here, $F_{p,x}$ is the resultant pressure force on the control surface, including any effect of the curved streamline boundaries. If the control surface is chosen so that this term can be neglected, $F_{p,x} \approx 0$, and solving for the drag gives

$$D' = \rho \left(U_\infty^2 h_1 - \int_{y_1}^{y_2} u^2\, dy \right). \tag{12}$$

The upstream height h_1 is not measured so it is eliminated using continuity. Multiplying Eq. (10) by U_∞ gives $U_\infty^2 h_1 = U_\infty \int u\, dy$, and substituting this into the expression above leaves only downstream quantities:

$$D' = \rho \int_{y_1}^{y_2} \left(U_\infty u - u^2 \right) dy = \rho \int_{y_1}^{y_2} u(U_\infty - u)\, dy. \tag{13}$$

For a unit span section, $D' = \frac{1}{2}\rho U_\infty^2 c C_{d,0}$. Dividing both sides by this dynamic pressure and chord gives

$$C_{d,0} = \frac{2}{c} \int_{y_1}^{y_2} \left(\frac{u}{U_\infty} \right) \left(1 - \frac{u}{U_\infty} \right) dy. \tag{14}$$

The derivation also identifies the limitation of Eq. (5). If the pressure force is not negligible, the neglected term remains in the momentum balance, and the present wake measurements cannot confirm that the static pressure has recovered.

B.2 Density and Density Uncertainty

The lab air temperature is 89°F, or 304.817 K. With an atmospheric pressure of 96,400 Pa and a gas constant of 287.05 J/(kg K), the ideal-gas law gives

$$\rho = \frac{P}{RT} = \frac{96{,}400}{287.05(304.817)} = 1.101744 \text{ kg/m}^3. \tag{15}$$

Using standard uncertainties of 100 Pa for pressure and 1 K for temperature, taken provisionally from Lab 1, and combining them in quadrature,

$$\sigma_\rho = \rho \sqrt{\left(\frac{\sigma_P}{P} \right)^2 + \left(\frac{\sigma_T}{T} \right)^2} = 0.003791 \text{ kg/m}^3. \tag{16}$$

These inputs must be verified against the Lab 3 equipment specifications before they are treated as confirmed instrument uncertainties.

B.3 Zero-Bias Treatment and Calibration Example

The pressure transducer has a zero bias of −2.3 Pa, which the acquisition software removes before export. For the calibration reading at 3.501 V, the exported dynamic pressure is 678.175 Pa and the exported reference velocity is 35.043 m/s. Both values are already corrected and are used without a second bias subtraction. The fitted calibration predicts

$$U_{\text{fit}} = \exp(-11.900193)(3.501)^{12.334578} = 35.010 \text{ m/s}, \tag{17}$$

so the measured velocity is 0.033 m/s above the fitted value at this point.

B.4 Traverse Position and Composite Trapezoidal Rule

A motor position of 5400 steps corresponds to

$$y = 5400\frac{0.0254}{6400} = 0.02143125 \text{ m} = 2.143125 \text{ cm}. \tag{18}$$

At that position behind the NACA 0012, the calibrated downstream velocity is 27.8843 m/s. With an upstream reference of 35.3051 m/s, the momentum-deficit integrand is

$$f_{5400} = \frac{27.8843}{35.3051}\left(1 - \frac{27.8843}{35.3051}\right) \approx 0.166010. \tag{19}$$

The preceding point, at 5300 steps, has an integrand of 0.164356, and the spacing between the two points is 100 steps, or 0.000396875 m. Their trapezoidal contribution is

$$\Delta I = \frac{0.164356 + 0.166010}{2}(0.000396875) \approx 6.55569 \times 10^{-5} \text{ m}. \tag{20}$$

Summing these contributions over all 48 positions gives a momentum integral of 0.000125807 m. For the 100 mm chord,

$$C_{d,0} = \frac{2(0.000125807)}{0.100} = 0.002516. \tag{21}$$

The same procedure for the NACA 4412 gives a momentum integral of −0.002242861 m and a drag coefficient of −0.044857. Both results keep the signs produced by the measured profiles.

Appendix C: MATLAB Code

The script below performs every calculation in this report. Sections 1 through 8 carry out the prescribed full-traverse reduction. Section 9 computes the regional contributions in Table 1, the wake-bounded estimates and their limit sensitivity in Tables 2 and A-5, and the temperature-correction sensitivity discussed in Section IV.G, using the local function `wakeOnlyCd`.

```
%% Lab 3 - Drag from airfoil wake measurements
% NACA 0012 and NACA 4412, Tuesday 6:00 group
%
% This is the wake momentum deficit calc from lab. I kept the pressure bias
% correction as its own switch so it's easy to see what it actually changes.
% The negative 4412 Cd stays in the results. I'm not deleting points to
% make it look nicer, section 9 at the bottom is where I figured out why
% it goes negative.
%
% NOTE ABOUT THE PRESSURE BIAS:
% The files don't say whether the software already removed the -2.3 Pa
% offset. I originally assumed it DIDN'T and asked in the discord.
% ** UPDATE: it's already removed by the software ** so biasAlreadyRemoved
% is set to true below and nothing gets subtracted twice.
%
% The calibration model is the one given in lab:
%   ln(U) = a ln(V) + b,  or  U = exp(b)*V^a
% It's an empirical power law fit, not King's law in its original form.
clear; clc; close all;

%% 1. Files, values, and recorded conditions
folder = fileparts(mfilename('fullpath'));
if isempty(folder), folder = pwd; end
outFolder = fullfile(folder, '5Lab3_Final_Results');
if ~exist(outFolder, 'dir'), mkdir(outFolder); end

calFile = fullfile(folder, 'Tues_A_600_A.txt');
file12  = fullfile(folder, 'Tues_A_600_0012.txt');
file44  = fullfile(folder, 'Tues_A_600_4412.txt');

chord = 0.100;                  % m, same for both airfoils
stepsPerRev = 400;
revPerInch = 16;
metersPerStep = 0.0254/(stepsPerRev*revPerInch);

Pambient = 96.4e3;              % Pa, what we recorded in lab
T_F = 89;                       % deg F, what we recorded in lab
T_C = (T_F - 32)*5/9;
T_K = T_C + 273.15;
Rair = 287.05;                  % J/(kg K)

useStandardDensity = false;     % only for if we hadn't recorded P and T,
% but we did, so this never runs. leaving it in for completeness.

pressureBias = -2.3;            % Pa, measured pressure zero offset
biasAlreadyRemoved = true;      % confirmed on discord, the export is already corrected

% Ambient instrument uncertainties from Lab 1. These only apply if the same
% equipment/specs were used in Lab 3, which I'm pretty sure is the case.
% nina if that's wrong pls dont make me lose points
sigmaP = 100;                   % Pa
sigmaT = 1.0;                   % K

% We weren't given hot-wire voltage/temperature uncertainties or traverse
% accuracy, so I'm not making up a total Cd error bar.

DPI = 300;

%% 2. Load the files and sanity check the data
% Calibration columns: temperature, dynamic pressure, hot-wire V, velocity
% Wake columns: temperature, dynamic pressure, hot-wire V, static drop,
%               upstream air velocity, motor position in steps
cal = readmatrix(calFile, 'FileType','text', 'Delimiter','\t', 'NumHeaderLines',1);
d12 = readmatrix(file12,  'FileType','text', 'Delimiter','\t', 'NumHeaderLines',1);
d44 = readmatrix(file44,  'FileType','text', 'Delimiter','\t', 'NumHeaderLines',1);

assert(size(cal,2)==4, 'The calibration file should have 4 columns.');
assert(size(d12,2)==6 && size(d44,2)==6, 'The wake files should have 6 columns.');
```

```
assert(all(isfinite(cal(:))) && all(isfinite(d12(:))) && all(isfinite(d44(:))), ...
    'One of the files has blank or invalid measurements');
assert(all(cal(:,2)>0) && all(d12(:,2)>0) && all(d44(:,2)>0), ...
    'Dynamic pressures need to be positive for the bias calculation');
assert(all(cal(:,3)>0) && all(cal(:,4)>0), ...
    'Calibration voltage and velocity need to be positive for a log-log fit');
assert(all(diff(d12(:,6))>0) && all(diff(d44(:,6))>0), ...
    'Motor positions have to increase during the traverse, check the files');

%% 3. Density and uncertainty
if useStandardDensity
    rho = 1.225;
    % Standard sea-level conditions, ONLY used to get a relative
    % uncertainty like the TA said for the standard-density option.
    % again, not actually running this, just here for technical accuracy
    % #ilovelarping
    P_forUncertainty = 101325;
    T_forUncertainty = 288.15;
    densitySource = 'TA allowed standard sea-level density';
else
    rho = Pambient/(Rair*T_K);
    P_forUncertainty = Pambient;
    T_forUncertainty = T_K;
    densitySource = 'Recorded pressure and temperature';
end

% rho = P/(RT), so the pressure and temperature terms add in RSS
sigmaRho = rho*sqrt((sigmaP/P_forUncertainty)^2 + ...
                    (sigmaT/T_forUncertainty)^2);

% Sutherland's law for viscosity, just need it for Reynolds number
mu = 1.716e-5*(T_K/273.15)^1.5*(273.15+110.4)/(T_K+110.4);

TableA1 = table({densitySource},Pambient,T_C,T_K,rho,sigmaP,sigmaT,sigmaRho, ...
    'VariableNames',{'DensityBasis','AmbientPressure_Pa','AmbientTemp_C', ...
    'AmbientTemp_K','Density_kg_m3','sigmaPressure_Pa', ...
    'sigmaTemp_K','sigmaDensity_kg_m3'});
writetable(TableA1, fullfile(outFolder,'Table_A1_Density.csv'));

%% 4. Pressure bias and hot-wire calibration
% q_corrected = q_recorded - q_bias. Since the bias is -2.3 Pa, this would
% ADD 2.3 Pa to each uncorrected reading.
% The air velocity column comes from the pressure reading, so instead of
% inventing a new ambient condition for each row I just scale it:
%   U_corrected = U_exported * sqrt(q_corrected/q_recorded)
% Since the export is already corrected, these end up identical.
qCalRaw = cal(:,2);
qCal = applyPressureBias(qCalRaw,pressureBias,biasAlreadyRemoved);
Vcal = cal(:,3);
UcalFile = cal(:,4);
Ucal = UcalFile.*sqrt(qCal./qCalRaw);

fit = polyfit(log(Vcal),log(Ucal),1);
a = fit(1);
b = fit(2);
UcalFit = exp(b)*Vcal.^a;
residual = Ucal-UcalFit;
R2log = 1-sum((log(Ucal)-log(UcalFit)).^2) / ...
          sum((log(Ucal)-mean(log(Ucal))).^2);
RMSE = sqrt(mean(residual.^2));

TableA2 = table(cal(:,1),qCalRaw,qCal,Vcal,UcalFile,Ucal,UcalFit,residual, ...
    'VariableNames',{'Temperature_C','DynamicPressure_Export_Pa', ...
    'DynamicPressure_Corrected_Pa','HotwireOutput_V', ...
    'Velocity_Export_mps','Velocity_Corrected_mps', ...
    'Velocity_Fit_mps','FitResidual_mps'});
writetable(TableA2, fullfile(outFolder,'Table_A2_Hotwire_Calibration.csv'));

% Keeping the original export-based fit too so the comparison is transparent
fitOriginal = polyfit(log(Vcal),log(UcalFile),1);

fig = figure('Color','w');
Vplot = linspace(min(Vcal),max(Vcal),300);
plot(Vcal,Ucal,'ko','MarkerFaceColor','k','MarkerSize',5); hold on;
plot(Vplot,exp(b)*Vplot.^a,'b-','LineWidth',1.7);
xlabel('Hot-wire output (V)'); ylabel('Air velocity (m/s)');
```

```
title('Hot-wire calibration'); grid on;
legend('Pressure-corrected measurements','Log-log regression','Location','northwest');
saveFigure(fig,outFolder,'Fig_01_Calibration',DPI);

fig = figure('Color','w');
plot(log(Vcal),log(Ucal),'ko','MarkerFaceColor','k'); hold on;
plot(log(Vplot),a*log(Vplot)+b,'b-','LineWidth',1.7);
xlabel('ln(hot-wire voltage / 1 V)');
ylabel('ln(velocity / 1 m/s)');
title(sprintf('Log-log calibration, R^2 = %.5f',R2log)); grid on;
legend('Measurements','Linear regression','Location','northwest');
saveFigure(fig,outFolder,'Fig_02_Log_Log_Calibration',DPI);

fig = figure('Color','w');
plot(Vcal,residual,'ko-','MarkerFaceColor','k'); hold on;
yline(0,'k--');
xlabel('Hot-wire output (V)'); ylabel('Measured - fitted velocity (m/s)');
title('Calibration residuals'); grid on;
saveFigure(fig,outFolder,'Fig_03_Calibration_Residuals',DPI);

%% 5. Wake velocities and drag coefficients
% TA Kaita said to take the MEAN of the upstream Air Velocity column in each
% Part B file for control surface 1. The upstream Pitot is fixed, it doesn't
% get traversed along y, so that's what I do here.
% y comes from the ACTUAL step counts. A move of 97 or 103 steps counts as
% 97 or 103 steps, not rounded to the nominal increment.
calRange = [min(Vcal),max(Vcal)];
r12 = wakeAnalysis(d12,'0012',a,b,chord,metersPerStep,rho,mu, ...
    pressureBias,biasAlreadyRemoved,calRange);
r44 = wakeAnalysis(d44,'4412',a,b,chord,metersPerStep,rho,mu, ...
    pressureBias,biasAlreadyRemoved,calRange);
results = {r12,r44};

% These tables are the measured profile data plus the calculations.
% No points are deleted or changed to make Cd positive (even though I
% really wanted to)
for j = 1:2
    r = results{j};
    if j==1, tableNumber = 'A3'; else, tableNumber = 'A4'; end

    Tprofile = table(r.steps,r.y_cm,r.yOverH,r.temperature, ...
        r.qRaw,r.qCorrected,r.voltage,r.UupFile,r.UupCorrected, ...
        r.Udown,r.UupNormalized,r.UdownNormalized,r.momentumTerm, ...
        'VariableNames',{'Position_steps','Position_cm','y_over_h', ...
        'Temperature_C','DynamicPressure_Export_Pa', ...
        'DynamicPressure_Corrected_Pa','HotwireOutput_V', ...
        'UpstreamVelocity_Export_mps','UpstreamVelocity_Corrected_mps', ...
        'DownstreamVelocity_mps','UpstreamOverMean', ...
        'DownstreamOverMean','MomentumIntegrand'});
    writetable(Tprofile,fullfile(outFolder, ...
        ['Table_' tableNumber '_NACA_' r.name '_Wake.csv']));

    % Dimensional profile: upstream reference and measured downstream wake
    fig = figure('Color','w');
    plot(r.UupCorrected,r.y_cm,'k.','MarkerSize',10); hold on;
    plot(r.Udown,r.y_cm,'bo-','LineWidth',1.25,'MarkerSize',4);
    xline(r.Uinf,'k--','LineWidth',1.3);
    xlabel('Air velocity (m/s)'); ylabel('Vertical traverse position (cm)');
    title(['NACA ' r.name ' - dimensional velocity profile']); grid on;
    legend('Upstream Pitot (not traversed)','Downstream hot wire', ...
        'Mean upstream reference','Location','best');
    saveFigure(fig,outFolder,['Fig_' r.name '_Dimensional_Profile'],DPI);

    % Height normalized by the measured traverse, not chord or span
    fig = figure('Color','w');
    plot(r.UupNormalized,r.yOverH,'k.','MarkerSize',10); hold on;
    plot(r.UdownNormalized,r.yOverH,'bo-','LineWidth',1.25,'MarkerSize',4);
    xline(1,'k--','LineWidth',1.3);
    xlabel('Velocity / mean upstream velocity'); ylabel('y / h');
    title(['NACA ' r.name ' - normalized velocity profile']); grid on;
    legend('Upstream Pitot (not traversed)','Downstream hot wire', ...
        'Upstream reference','Location','best');
    saveFigure(fig,outFolder,['Fig_' r.name '_Normalized_Profile'],DPI);

    fig = figure('Color','w');
    plot(r.momentumTerm,r.y_cm,'bo-','LineWidth',1.2,'MarkerSize',4); hold on;
```

```
    xline(0,'k--');
    xlabel('(u/Uinf)(1 - u/Uinf)'); ylabel('Vertical position (cm)');
    title(['NACA ' r.name ' - momentum-deficit integrand']); grid on;
    saveFigure(fig,outFolder,['Fig_' r.name '_Momentum_Integrand'],DPI);

    % Temperature drifted during the traverse, so I'm saving this as evidence
    % for the discussion. It is NOT an automatic temperature correction.
    fig = figure('Color','w');
    plot(r.y_cm,r.temperature,'ro-','MarkerSize',4,'LineWidth',1.2);
    xlabel('Traverse position (cm)'); ylabel('Measured air temperature (deg C)');
    title(['NACA ' r.name ' - temperature during traverse']); grid on;
    saveFigure(fig,outFolder,['Fig_' r.name '_Temperature'],DPI);
end

%% 6. Comparison with the uncorrected calculation
% Wanted to show exactly how much the -2.3 Pa correction changes things.
% 'As exported' uses the velocity columns and the original log-log fit,
% same as the first version of this script.
raw12 = wakeAnalysis(d12,'0012',fitOriginal(1),fitOriginal(2), ...
    chord,metersPerStep,rho,mu,pressureBias,true,calRange);
raw44 = wakeAnalysis(d44,'4412',fitOriginal(1),fitOriginal(2), ...
    chord,metersPerStep,rho,mu,pressureBias,true,calRange);

% Lab 2 pressure-only numbers are here for context, NOT as a direct
% measurement of the Lab 3 total zero-lift drag.
% Also the 4412 Lab 2 value is at 0 deg, not its negative zero-lift angle.
Airfoil = {'NACA 0012';'NACA 4412'};
UinfExport_mps = [mean(d12(:,5));mean(d44(:,5))];
UinfUsed_mps = [r12.Uinf;r44.Uinf];
ReChord = [r12.Re;r44.Re];
WakeIntegral_m = [r12.integral;r44.integral];
DragPerSpan_N_per_m = [r12.Dprime;r44.Dprime];
CdAsExported = [raw12.Cd;raw44.Cd];
CdWithSelectedBiasTreatment = [r12.Cd;r44.Cd];
PositiveCdContribution = [r12.CdPositive;r44.CdPositive];
NegativeCdContribution = [r12.CdNegative;r44.CdNegative];
Lab2PressureCd_at_0deg = [0.0151;0.0242];
AbbottCd = [0.006;0.006];   % approx, read off Abbott App. IV (smooth, Re = 3e6, cl = 0)

TableA5 = table(Airfoil,UinfExport_mps,UinfUsed_mps,ReChord, ...
    WakeIntegral_m,DragPerSpan_N_per_m,CdAsExported, ...
    CdWithSelectedBiasTreatment,PositiveCdContribution, ...
    NegativeCdContribution,Lab2PressureCd_at_0deg,AbbottCd);
writetable(TableA5,fullfile(outFolder,'Table_A5_Drag_Results.csv'));

%% 7. Reference-velocity sensitivity (diagnostic only)
% This does NOT replace Kaita's mean upstream velocity. Just checking how
% much the result moves if that reference speed were a little off.
sensitivity = table();
for j = 1:2
    r = results{j};
    trialU = r.Uinf + (-2:0.25:2)';
    trialCd = zeros(size(trialU));
    for k = 1:length(trialU)
        ratio = r.Udown/trialU(k);
        trialCd(k) = (2/chord)*trapz(r.y_m,ratio.*(1-ratio));
    end
    one = table(repmat({['NACA ' r.name]},numel(trialU),1), ...
        trialU,repmat(r.Uinf,numel(trialU),1),trialCd, ...
        'VariableNames',{'Airfoil','TrialReferenceVelocity_mps', ...
        'MeasuredMeanReference_mps','CalculatedCd'});
    sensitivity = [sensitivity;one]; %#ok<AGROW>

    fig = figure('Color','w');
    plot(trialU,trialCd,'b-','LineWidth',1.6); hold on;
    xline(r.Uinf,'k--','LineWidth',1.2);
    yline(0,'k:');
    plot(r.Uinf,r.Cd,'ko','MarkerFaceColor','k');
    xlabel('Assumed upstream reference velocity (m/s)');
    ylabel('Calculated Cd');
    title(['NACA ' r.name ' - sensitivity to upstream reference']); grid on;
    legend('Sensitivity curve','Measured mean upstream', ...
        'Zero drag','Primary calculation','Location','best');
    saveFigure(fig,outFolder,['Fig_' r.name '_Reference_Sensitivity'],DPI);
end
writetable(sensitivity,fullfile(outFolder,'Diagnostic_Reference_Sensitivity.csv'));
```

```
297
298 %% 8. Print the results and save the workspace
299 fprintf('\n LAB 3 - TUESDAY GROUP \n');
300 fprintf('Chord for both models          = %.3f m\n',chord);
301 fprintf('Motor conversion               = %.9f m/step\n',metersPerStep);
302 fprintf('14000 steps                    = %.4f cm\n',14000*metersPerStep*100);
303 fprintf('Density used                   = %.5f +/- %.5f kg/m^3\n',rho,sigmaRho);
304 fprintf('Recorded pressure bias         = %+.2f Pa\n',pressureBias);
305 if biasAlreadyRemoved
306     fprintf('Bias setting                   = already removed by software\n');
307 else
308     fprintf('Bias setting                   = assumed NOT removed by software\n');
309     fprintf('Pressure treatment             = q_corrected = q_exported - (%.2f) Pa\n',pressureBias);
310 end
311 fprintf('Calibration                    = U = exp(%.6f) * V^(%.6f)\n',b,a);
312 fprintf('Calibration R^2 (log)          = %.6f\n',R2log);
313 fprintf('Calibration RMSE               = %.4f m/s\n',RMSE);
314
315 for j = 1:2
316     r = results{j};
317     if j==1, oldCd = raw12.Cd; else, oldCd = raw44.Cd; end
318     fprintf('\nNACA %s (%d measurements)\n',r.name,numel(r.steps));
319     fprintf('  Mean upstream, exported       = %.4f m/s\n',r.UinfFile);
320     fprintf('  Mean upstream, used           = %.4f m/s\n',r.Uinf);
321     fprintf('  Downstream velocity range     = %.3f to %.3f m/s\n', ...
322         min(r.Udown),max(r.Udown));
323     fprintf('  Measured traverse height      = %.4f cm\n',r.y_m(end)*100);
324     fprintf('  Points faster than upstream   = %d / %d\n', ...
325         sum(r.Udown>r.Uinf),numel(r.steps));
326     fprintf('  Outside calibration range     = %d / %d\n', ...
327         r.outsideCount,numel(r.steps));
328     fprintf('  Positive contribution to Cd   = %+.6f\n',r.CdPositive);
329     fprintf('  Negative contribution to Cd   = %+.6f\n',r.CdNegative);
330     fprintf('  Cd from data as exported      = %+.6f\n',oldCd);
331     fprintf('  Cd with chosen bias treatment = %+.6f\n',r.Cd);
332     fprintf('  Drag per unit span            = %+.5f N/m\n',r.Dprime);
333     fprintf('  Uinf making this integral zero = %.3f m/s (diagnostic only)\n', ...
334         r.Uzero);
335     if r.Cd<0
336         fprintf('  FLAG: negative Cd from the simplified wake equation.\n');
337         fprintf('  Keeping it in the report and discussing the assumptions.\n');
338     end
339 end
340 fprintf('\nLab 2 comparison uses pressure-only drag at 0 deg.\n');
341 fprintf('Lab 3 4412 was supposed to be near its negative zero-lift angle.\n');
342 fprintf('No total Cd uncertainty assigned without hot-wire/position specs.\n');
343 fprintf('Saved tables and figures to: %s\n',outFolder);
344
345 save(fullfile(outFolder,'Lab3_Analysis_Workspace.mat'), ...
346     'cal','d12','d44','r12','r44','raw12','raw44','fit', ...
347     'R2log','RMSE','rho','sigmaRho','TableA1','TableA2','TableA5', ...
348     'pressureBias','biasAlreadyRemoved','metersPerStep');
349
350 %% 9. Wake-bounded integration, regional breakdown, temperature check
351 % This is the part where I figured out WHY the 4412 goes negative.
352 % The prescribed full-traverse Cd from section 5 is NOT changed here.
353 % The derivation assumes u = Uinf at the edges of the control volume, but
354 % our outer flow doesn't do that, so these are diagnostics showing where
355 % the total comes from and what the wake by itself gives.
356
357 % 9a. Split the full-traverse Cd up by region (mean upstream reference)
358 % nina ignore the printouts here, I was just checking the regions add back
359 % up to the section 5 totals (they do)
360 regions12 = [0 3600; 3600 4000; 4000 6800; 6800 14002];
361 regions44 = [0 2400; 2400 4604; 4604 6800; 6800 14000];
362 fprintf('\nREGIONAL CONTRIBUTIONS TO Cd (mean upstream reference)\n');
363 for k = 1:size(regions12,1)
364     fprintf('  NACA 0012, steps %5d to %5d : %+.4f\n', regions12(k,1), ...
365         regions12(k,2), wakeOnlyCd(r12,regions12(k,1),regions12(k,2),chord,'pitot'));
366 end
367 for k = 1:size(regions44,1)
368     fprintf('  NACA 4412, steps %5d to %5d : %+.4f\n', regions44(k,1), ...
369         regions44(k,2), wakeOnlyCd(r44,regions44(k,1),regions44(k,2),chord,'pitot'));
370 end
371
372 % 9b. Wake-bounded estimates, limits sit at the flat parts on each side
```

```
Cd12_wakeUp   = wakeOnlyCd(r12,4000,6800,chord,'pitot');  % 1.59 to 2.70 cm
Cd12_wakeEdge = wakeOnlyCd(r12,4000,6800,chord,'edge');
Cd44_wakeUp   = wakeOnlyCd(r44,4604,6800,chord,'pitot');  % 1.83 to 2.70 cm
Cd44_wakeEdge = wakeOnlyCd(r44,4604,7200,chord,'edge');   % 1.83 to 2.86 cm
fprintf('\nWAKE-BOUNDED Cd\n');
fprintf('  NACA 0012: ue = Uinf %.4f | ue from wake edges %.4f\n', ...
    Cd12_wakeUp, Cd12_wakeEdge);
fprintf('  NACA 4412: ue = Uinf %.4f | ue from wake edges %.4f\n', ...
    Cd44_wakeUp, Cd44_wakeEdge);

% 9c. How much do the answers move if I pick the limits differently?
% Only moving them along the flat plateaus next to the wake. The 0012 has
% no plateau below 4000 steps because of that weird step at 1.43 cm, so
% only its top limit moves.
Cd12_range = [];
for hi = [6600 6800 7000 7204 7400]
    Cd12_range = [Cd12_range, wakeOnlyCd(r12,4000,hi,chord,'pitot'), ...
                              wakeOnlyCd(r12,4000,hi,chord,'edge')]; %#ok<AGROW>
end
Cd44_range = [];
for lo = [4003 4401 4604]
    Cd44_range = [Cd44_range, wakeOnlyCd(r44,lo,6800,chord,'pitot')]; %#ok<AGROW>
    for hi = [7200 7420 7600]
        Cd44_range = [Cd44_range, wakeOnlyCd(r44,lo,hi,chord,'edge')]; %#ok<AGROW>
    end
end
fprintf('  Limit sensitivity, NACA 0012: %.4f to %.4f\n', min(Cd12_range), max(Cd12_range));
fprintf('  Limit sensitivity, NACA 4412: %.4f to %.4f\n', min(Cd44_range), max(Cd44_range));

% 9d. What a uniform 1 percent hot-wire bias does to the full-traverse Cd
% (spoiler, a lot)
fprintf('\nUNIFORM 1 PERCENT HOT-WIRE BIAS\n');
for j = 1:2
    r = results{j};
    ratio = 1.01*r.Udown/r.Uinf;
    fprintf('  NACA %s: Cd %.4f -> %.4f\n', r.name, r.Cd, ...
        (2/chord)*trapz(r.y_m,ratio.*(1-ratio)));
end

% 9e. Temperature correction sensitivity (NOT used in the final results)
% Standard CTA correction: V_corr = V*sqrt((Tw - Tref)/(Tw - Ta))
% We never recorded the wire temperature Tw, so I just tried three values.
% Calibration and wake voltages both get corrected to Tref, then refit.
% nina this is only here to back up the discussion section, I know I can't
% actually apply it without the overheat setting
Tref = 30; % deg C
fprintf('\nTEMPERATURE-CORRECTION SENSITIVITY (assumed wire temperature)\n');
for Tw = [200 250 300]
    tc = @(V,Ta) V.*sqrt((Tw-Tref)./(Tw-Ta));
    pT = polyfit(log(tc(Vcal,cal(:,1))),log(Ucal),1);
    CdT = zeros(1,2);
    for j = 1:2
        r = results{j};
        uT = exp(pT(2))*tc(r.voltage,r.temperature).^pT(1);
        CdT(j) = (2/chord)*trapz(r.y_m,(uT/r.Uinf).*(1-uT/r.Uinf));
    end
    fprintf('  Tw = %3d C: Cd 0012 = %.4f, Cd 4412 = %.4f\n', Tw, CdT(1), CdT(2));
end

%% Local functions
function qCorrected = applyPressureBias(qMeasured,bias,alreadyRemoved)
if alreadyRemoved
    qCorrected = qMeasured;
else
    qCorrected = qMeasured-bias;
end
assert(all(qCorrected>0), ...
    'Bias correction produced nonpositive dynamic pressure.');
end

function r = wakeAnalysis(data,name,a,b,chord,metersPerStep,rho,mu, ...
    pressureBias,biasAlreadyRemoved,calRange)
r.name = name;
r.temperature = data(:,1);
r.qRaw = data(:,2);
r.qCorrected = applyPressureBias(r.qRaw,pressureBias,biasAlreadyRemoved);
```

```
r.voltage = data(:,3);
r.steps = data(:,6);

r.UupFile = data(:,5);
r.UupCorrected = r.UupFile.*sqrt(r.qCorrected./r.qRaw);
r.UinfFile = mean(r.UupFile);
r.Uinf = mean(r.UupCorrected);          % mean at CS1, per the TA
r.Udown = exp(b)*r.voltage.^a;

r.y_m = (r.steps-r.steps(1))*metersPerStep;
r.y_cm = 100*r.y_m;
r.yOverH = r.y_m/r.y_m(end);
r.UupNormalized = r.UupCorrected/r.Uinf;
r.UdownNormalized = r.Udown/r.Uinf;

r.momentumTerm = r.UdownNormalized.*(1-r.UdownNormalized);
r.integral = trapz(r.y_m,r.momentumTerm);
r.Cd = (2/chord)*r.integral;
r.Dprime = rho*r.Uinf^2*r.integral;
r.Re = rho*r.Uinf*chord/mu;

% Split the integral into its positive and negative areas. If a segment
% crosses zero, it gets split right where the line passes through zero.
[positiveArea,negativeArea] = signedTrapezoids(r.y_m,r.momentumTerm);
r.CdPositive = (2/chord)*positiveArea;
r.CdNegative = (2/chord)*negativeArea;
% nina ignore this assert, just me making sure the split adds back up
assert(abs(r.Cd-(r.CdPositive+r.CdNegative))<1e-10, ...
    'Signed integration check did not match the total integral.');

% The reference speed that would make this exact wake integrate to zero.
% Only a sensitivity check, not a correction I'm proposing.
r.Uzero = trapz(r.y_m,r.Udown.^2)/trapz(r.y_m,r.Udown);
r.outsideCount = sum(r.voltage<calRange(1) | r.voltage>calRange(2));
end

function [positiveArea,negativeArea] = signedTrapezoids(y,f)
positiveArea = 0;
negativeArea = 0;
for k = 1:length(y)-1
    dy = y(k+1)-y(k);
    f1 = f(k); f2 = f(k+1);
    if f1*f2<0
        % this segment crosses zero between the two points
        length1 = dy*abs(f1)/(abs(f1)+abs(f2));
        area1 = 0.5*f1*length1;
        area2 = 0.5*f2*(dy-length1);
        positiveArea = positiveArea + max(area1,0) + max(area2,0);
        negativeArea = negativeArea + min(area1,0) + min(area2,0);
    else
        area = 0.5*(f1+f2)*dy;
        positiveArea = positiveArea+max(area,0);
        negativeArea = negativeArea+min(area,0);
    end
end
end

function Cd = wakeOnlyCd(r,s0,s1,chord,mode)
% Integrates the momentum deficit ONLY between motor positions s0 and s1.
% 'pitot' -> reference is the mean upstream velocity (r.Uinf)
% 'edge'  -> reference goes linearly between the hot-wire readings at the
%            two limits, so any uniform hot-wire bias cancels out
k = r.steps >= s0 & r.steps <= s1;
y = r.y_m(k);
u = r.Udown(k);
if strcmp(mode,'edge')
    ue = interp1([y(1) y(end)],[u(1) u(end)],y);
else
    ue = r.Uinf*ones(size(u));
end
Cd = (2/chord)*trapz(y,(u./ue).*(1-u./ue));
end

function saveFigure(fig,folder,name,dpi)
set(fig,'Position',[100 100 750 500]);
set(findall(fig,'Type','axes'),'FontName','Times New Roman','FontSize',11);
```

```
525 exportgraphics(fig,fullfile(folder,[name '.png']), ...
526     'Resolution',dpi,'BackgroundColor','white');
527 close(fig);
528 end
```

Acknowledgments

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References

[1] Arizona State University, Department of Mechanical and Aerospace Engineering, "Experiment 3: Drag Coefficients for the NACA 0012 and 4412 from Wake-Profile Measurements," laboratory handout supplied for the 2026 course and Tuesday 6:00 section measurement data.

[[2]] Barlow, J. B., Rae, W. H., Jr., and Pope, A., *Low-Speed Wind Tunnel Testing*, 3rd ed., Wiley, New York, 1999.

[3] Bruun, H. H., *Hot-Wire Anemometry: Principles and Signal Analysis*, Oxford University Press, Oxford, 1995. https://doi.org/10.1093/oso/9780198563426.001.0001

[4] Abbott, I. H., and von Doenhoff, A. E., *Theory of Wing Sections, Including a Summary of Airfoil Data*, Dover Publications, New York, 1959. *[Add the page numbers of the NACA 0012 and NACA 4412 section-data figures used in Table A-5.]*

[5] Bowes, D., "Calculation of Lift and Drag from Surface Pressure Measurements on NACA 0012 and NACA 4412 Airfoils," AEE 361 Lab 2 report, Arizona State University, 2026 (unpublished course report).