Calculation of Lift and Drag from Surface Pressure Measurements on NACA 0012 and NACA 4412 Airfoils

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The lift and drag of a NACA 0012 and a NACA 4412 airfoil were found by integrating surface pressure coefficients measured in a low speed wind tunnel. Each airfoil had a 100 mm chord and was tested at a fixed tunnel setting of 40 Hz, which gave a freestream velocity of about 33.5 to 39.3 m/s and a chord Reynolds number of about 2×10^5. Surface pressures were read with a scannivalve and referenced to the test section static pressure, and the pressure coefficients were integrated around each airfoil to give lift and pressure drag coefficients over a range of angles of attack. Uncertainties were estimated by integrating inner and outer bounds on the pressure coefficient. For the NACA 0012, the lift slope was 6.49 per rad, about 3 percent above the thin airfoil value of 2π, the zero-lift angle was 0 deg, and the maximum lift coefficient was 1.015 at 10 deg, with stall between 10 and 12 deg. The maximum lift-to-drag ratio was 11.4 at 4 deg. For the NACA 4412, the measured lift slope was only 2.64 per rad and the zero-lift angle was -6.4 deg, against a thin airfoil zero-lift angle of -4.15 deg, and the pressure drag came out near zero or negative above 4 deg. One suspect pressure tap on the 4412 changed the lift coefficient by 0.10 to 0.13, which was much larger than the instrument uncertainty of 0.01 to 0.02 in lift coefficient. The results show that lift can be recovered from surface pressures with reasonable tap coverage, and that pressure-only drag and the 4412 results are limited by tap spacing near the leading edge and by one bad tap.

Nomenclature

c	=	airfoil chord length, m
C_a	=	axial force coefficient
C_d	=	drag coefficient (pressure drag only)
C_l	=	lift coefficient
C_n	=	normal force coefficient
C_l/C_d	=	lift-to-drag ratio
C_p	=	pressure coefficient
P_{amb}	=	ambient pressure, Pa
p	=	local static pressure at a tap, Pa
P_∞	=	freestream static pressure, Pa
q_∞	=	freestream dynamic pressure, Pa
R	=	specific gas constant of air (287.05), J/(kg·K)
Re_c	=	Reynolds number based on chord
r_i	=	scannivalve reading at tap i, in. H2O
r_0	=	scannivalve reading at port 0 (static reference), in. H2O
T	=	ambient air temperature, K
V_∞	=	freestream velocity, m/s
x, y	=	chordwise and vertical coordinates measured from the leading edge, m
α	=	angle of attack, deg

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α_{L0} = zero-lift angle of attack, deg
δ = uncertainty in the quantity that follows
μ = dynamic viscosity of air, Pa·s
ρ = air density, kg/m³

Subscripts
l, u = lower surface, upper surface
∞ = freestream

I. Introduction

The force on an airfoil comes from the pressure and shear stress the air applies over its surface. In lecture we have learned that for an aerodynamic shape at a moderate angle of attack, most of the lift and most of the drag that can be measured this way come from the pressure. That means the forces can be found by measuring the pressure at a set of points around the airfoil and integrating it over the surface, without the need to use a force balance. The pressure at each point is reported as a pressure coefficient, C_P which compares the local static pressure to the freestream static pressure and scales the difference by the freestream dynamic pressure:

$$C_p = \frac{p - p_\infty}{q_\infty} \tag{1}$$

$$q_\infty = \frac{1}{2}\rho V_\infty^2 \tag{2}$$

Some important characteristics of C_P should be noted before continuing. A C_P equal to 1 marks a stagnation point, and a negative C_P marks a region where the flow is faster than the freestream and the pressure is below freestream, this comes from Bernoulli, principle, meaning as local static pressure is lower (also denoted as a -C_P value) dynamic pressure needs to rise to keep Bernoulli's equation constant, dynamic pressure relies directly on velocity meaning that as local pressure lowers, local velocity will rise (for steady, incompressible flow). Bernoulli's equation relates the two, with $C_P = 1 - (V/V_\infty)^2$ meaning that a distribution of Cp is also a map of the local flow speed. People who use LLM's to do their homework tend to get the phrase "suction at peak" from this principle, because this action of low local static pressure creating a vacuum like effect to raise dynamic pressure is technically a "suction" method, and personally I do like to refer to it as that in my head, however I will not risk any academic integrity issues so for this lab report I will not be referring to and low C_P values as causing any "suction". This relation between C_P and local velocity compared to freestream velocity will be used in this experiment, as the tunnel in this experiment ran at about 34 to 39 m/s, (constant 40 Hz setting) this is a Mach number below 0.12, so treating the flow as incompressible is reasonable [1].

Pressure always acts normal to the surface, so the integrated pressure gives a force normal to the chord (C_n) and a force along the chord (C_a). The normal force comes mostly from the difference between the lower and upper surface pressure. The axial force comes from the pressure acting on the forward and rearward facing parts of the surface, which is why it depends on the surface slope. Both are found by integrating our found C_p values around the airfoil, and both are then rotated through the angle of attack α to give the lift and drag coefficients, which are defined relative to the freestream direction. The drag found this way is pressure drag only. Skin friction drag is not captured, and for an ideal, attached, inviscid flow the pressure drag would be zero (d'Alembert's paradox), so any pressure drag that is measured comes from viscous effects such as boundary layer growth and separation [1].

Thin airfoil theory gives a baseline for what the results should look like. It predicts a lift curve (slope of our C_l/α values) that is linear in angle of attack with a slope of 2π per radian (about 0.11 per degree), and a lift coefficient of

$$c_l = 2\pi\left(\alpha - \alpha_{L0}\right) \tag{3}$$

where α_{L0} is the zero lift angle of attack. For a symmetric airfoil, α_{L0} is zero, meaning that camber shifts the whole curve, for the NACA 4412 camber line (4 percent maximum camber at 40 percent chord), thin airfoil theory gives α_{L0} = -4.15 deg, so the airfoil produces lift at zero angle of attack (C_l = 0.455) and needs a negative angle to produce none. It's important to note that real lift curves stay linear only up to a point, and at higher angles the flow separates from the upper surface and the lift stops increasing and falls, which is called a stall, and the angle where this happens depends on the airfoil shape and the

Reynolds number, we will be witnessing this in our data. The lift to drag ratio is the measure of aerodynamic efficiency, and because drag grows faster than lift as the angle increases, its maximum generally sits at a lower angle than the angle of maximum lift [2].

In this experiment, two airfoils were tested to compare these effects. The NACA 0012, which is a symmetric airfoil with a thickness of 12 percent of the chord. And the NACA 4412, which has the same thickness but with a 4 percent camber instead of none in the 0012's case. Both models had a chord of 100 mm.

The objective of this experiment was to measure the pressure distribution around a NACA 0012 and a NACA 4412 airfoil over a range of angles of attack, and to find the lift and drag coefficients (C_l, C_d) and the lift to drag ratio (C_l/C_d) from those distributions. The measured lift slope, zero-lift angle, and stall behavior of each airfoil were compared with thin airfoil theory and with each other, and the uncertainty of the results was estimated.

II. Experimental Procedure

A. Equipment and Setup

The experiment was run in the ASU low speed subsonic wind tunnel in USE170, with a closed test section, at a fan setting of 40 Hz for every run. The tunnel freestream conditions were measured with a pitot static tube mounted in the test section, and its dynamic pressure was read by a pressure transducer and recorded in Pa, and the ambient temperature and pressure were recorded from the laboratory instruments. Fig. 1 shows the setup.

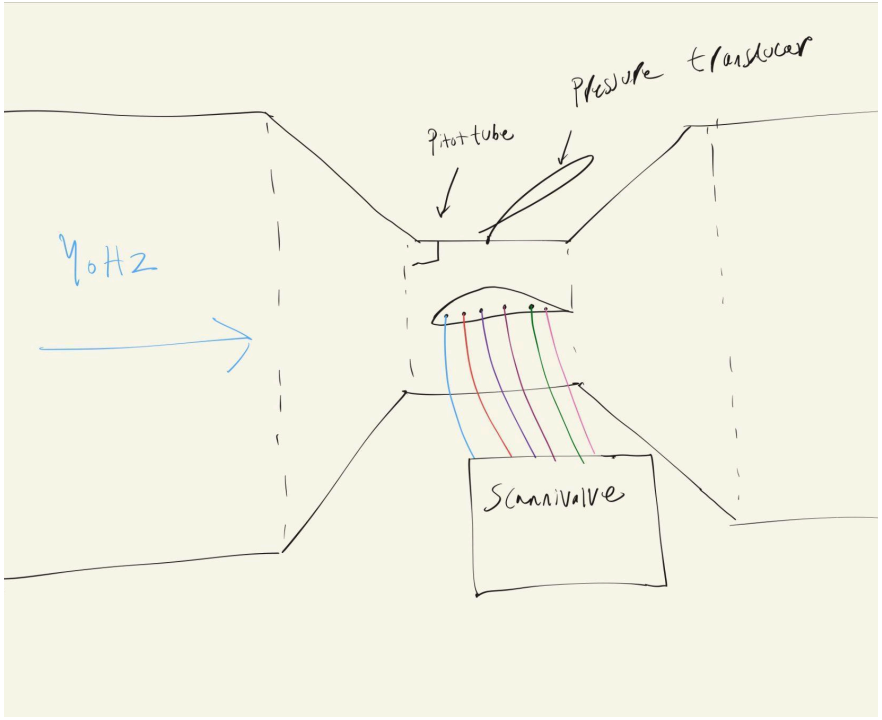


Fig. 1 Setup diagram of the low speed wind tunnel with airfoil inside connected to scannivalve at ports.

As mentioned in the introduction, two airfoil models were tested, a NACA 0012 and a NACA 4412, each with a chord of c = 100 mm and mounted across the test section on a pivot that was able to set the angle of attack. Each model had pressure taps along its surface that were connected by tubing to a scannivalve pressure scanner. The NACA 0012 had one row of 9 taps, as a symmetric airfoil will have the same values on the upper and lower surface, whereas The NACA 4412 had 8 taps on the upper surface and 7 on the lower surface. The tap locations were taken from the geometry handout provided with the lab handout [3]. For the 0012 they were given in mm from the leading edge and were divided by the chord to get x/c, and for the 4412 they were given directly as x/c and y/c. The 0012 surface coordinates were computed from the NACA thickness equation for a 12 percent thick symmetric airfoil [2, 3].

The scannivalve read one tap at a time by stepping through its ports in order. Port 0 was connected to the static pressure of the test section and was used as the reference pressure for all of the other ports. Ports 1 onward were connected to the airfoil taps in order. The scannivalve pressure transducer output was read in inches of water, and the port to tap map for each model

is listed in Table A-3 of Appendix A. The scannivalve could read a pressure no lower than -10.624 in. H2O, and any tap below this value was recorded at that limit.

B. Test Procedure

The following steps were carried out for each airfoil.

1. The model was mounted in the test section and its tubing was connected to the scannivalve, and the tunnel was set to 40 Hz.
2. The angle of attack was set with the model pivot and checked against the angle scale. The NACA 0012 was tested at -12, -10, -8, -4, 0, 4, 8, 10, and 12 deg, and the NACA 4412 was tested at 0 to 14 deg in steps of 2 deg.
3. After the flow had settled at each angle, the scannivalve was stepped through its ports, starting at port 0. At each port, the scannivalve reading, the pitot static dynamic pressure, the tunnel velocity, the ambient temperature, and the ambient pressure were recorded in the data file with the angle of attack and port number.
4. The angle was then changed and step 3 was repeated. The tunnel was kept at 40 Hz throughout, so the freestream velocity was not held constant and changed with the angle of attack from about 33.5 to 39.3 m/s as the model blockage and drag changed.

Only the upper surface taps of the NACA 0012 were measured. Because the airfoil is symmetric, the pressure on the lower surface at an angle α is the same as the pressure on the upper surface at -α, so the negative angles were used to give the lower surface. During the 4412 run at 14 deg, the scannivalve saturated on upper taps 1 and 2 and on lower tap 1, and those readings were not used. In addition, the reading from the 4412 upper surface tap at x/c = 0.61 (port 7) was close to the static reference at every angle, which was not consistent with the neighboring taps. The other members of the group reported the same behavior. The reading was kept in the main results and its effect is shown as a sensitivity in Sec. III.B.

C. Data Reduction

The density of the air was found from the ideal gas law using the recorded ambient pressure and temperature,

$$\rho = \frac{P_{amb}}{RT} \tag{4}$$

where $R = 287.05\,J/(kg * K)$. The dynamic pressure q_∞ was taken from the pitot static transducer and the freestream velocity was found from $V_\infty = \sqrt{\frac{2q_\infty}{\rho}}$. The Reynolds number based on chord was

$$Re_c = \frac{\rho V_\infty c}{\mu} \tag{5}$$

with the viscosity μ found from Sutherland's law [4].

The pressure at each tap relative to the freestream static pressure was found by subtracting the port 0 reading from the tap reading, and converting from inches of water to Pa with 249.09 Pa per in. H2O. This was divided by the dynamic pressure to give the pressure coefficient,

$$C_{p,i} = \frac{(r_i - r_0)(249.09)}{q_\infty} \tag{6}$$

where r_i is the reading at tap i and r_0 is the port 0 reading, both in in. H2O. These values were arranged by tap position for each angle of attack, and the pressure distributions were plotted against x/c with the upper surface and lower surface in separate colors and with the vertical axis reversed so that the side with negative C_p appears above the side with positive C_p (natural for me to do because when analyzing conventional wing airfoils at positive alpha values, upper surface is the negative C_p so the sign convention looks good. Lift and drag coefficients were then found by integrating the pressure coefficients, as described below.

All of the data reduction was done in a MATLAB script that read the three data files (one per airfoil surface) and produced every table and figure in this report. The script was run twice on the same data. In the main run every tap was used as measured. In the second run a switch in the script removed the anomalous NACA 4412 upper surface tap at x/c = 0.61 (port 7) before the extrapolation and integration, so that the effect of that one reading on the force coefficients could be compared directly (Table 1). Readings at the scannivalve limit were dropped in both runs. The pressure uncertainty for each tap was treated as a single term of 2.5 Pa, and not as two independent readings (tap and port 0) added in quadrature, following the method given in the lab discussion. The linear region used for the lift slope and zero-lift angle was set to -8 to 8 deg for the NACA 0012 and 0 to 8 deg for the NACA 4412, and the fitted values depend on those ranges.

D. Uncertainty and Force Integration

Pressure coefficients were computed at every tap from the scannivalve reading, the port 0 (test section static) reading, and the pitot dynamic pressure at the same angle of attack:

$$C_p = \frac{p_i - p_\infty}{q_\infty} \tag{7}$$

The port 0 reading was subtracted from each tap reading and the difference was converted from in. H2O to Pa with 249.09 Pa per in. H2O. Readings at or below the scannivalve saturation value (-10.62 in. H2O) were treated as invalid and left out of every calculation.

The uncertainty in each Cp was found by propagating the pressure and dynamic pressure uncertainties through the definition above:

$$\delta C_p = \sqrt{\left(\frac{\delta p}{q_\infty}\right)^2 + \left(\frac{C_p \delta q_\infty}{q_\infty}\right)^2} \tag{8}$$

The machine error chart gives instrument uncertainties in volts (0.005 V for the scannivalve and 0.01 V for the pressure transducer). These were converted to pressure with the Lab 1 calibration slopes of about 504 Pa/V and 1339 Pa/V, giving a scannivalve uncertainty of 2.5 Pa and a dynamic pressure uncertainty of 13.4 Pa.

To carry the Cp uncertainty into the force coefficients, two extra distributions were built at every angle of attack. The outer bound lowers the upper surface Cp by δCp and raises the lower surface Cp by δCp at every tap, which gives the largest enclosed area. The inner bound does the opposite and gives the smallest. The true, inner, and outer distributions were each integrated the same way, and the uncertainty in each coefficient was taken as half the spread of the three results:

$$\delta c_l = \frac{c_{l,max} - c_{l,min}}{2} \tag{9}$$

$$\delta c_d = \frac{c_{d,max} - c_{d,min}}{2} \tag{10}$$

Note that the same rule was used for C_l/C_d.

The taps do not reach the leading or trailing edge, so the measured distributions do not form closed curves. Each surface was extrapolated linearly from its two nearest taps to x/c = 0 and x/c = 1, and the two surfaces were forced to share one Cp value at each end. Without this, the force integrals would skip the leading edge region, where the largest magnitude of C_p is found. The normal and axial force coefficients were then found with the trapezoidal rule, using the airfoil coordinates from the geometry handout:

$$c_n = \int_0^1 \left(C_{p,l} - C_{p,u}\right) d\left(\frac{x}{c}\right) \tag{11}$$

$$c_a = \int C_{p,u}\, d\left(\frac{y_u}{c}\right) - \int C_{p,l}\, d\left(\frac{y_l}{c}\right) \tag{12}$$

and these were rotated through the angle of attack into lift and drag coefficients:

$$c_l = c_n \cos\alpha - c_a \sin\alpha \tag{13}$$

$$c_d = c_n \sin\alpha + c_a \cos\alpha \tag{14}$$

The NACA 0012 had only one row of taps. Its lower surface at angle of attack α was taken from the upper surface data recorded at -α, which is valid because the airfoil is symmetric.

III. Results

A. Pressure Distributions

The initial test conditions were similar for both airfoils, with an air temperature of about 304.5 K, an ambient pressure of 96.5 kPa, and a density of about 1.10 kg/m^3. As mentioned earlier, the tunnel was held at 40 Hz for every run, but the test section velocity still moved between about 33.5 and 39.3 m/s with angle of attack. The chord Reynolds number was about $2.1 * 10^5$ for the NACA 0012 and $2.2 * 10^5$ for the NACA 4412. The $-C_p$ distributions are shown in 13 plots, one per angle of attack: five for the NACA 0012 (0, 4, 8, 10, and 12 deg, Figs. 2–6) and eight for the NACA 4412 (0 to 14 deg in 2 deg steps, Figs. 7–14). Each plot shows the upper surface in blue, the lower surface in red, and the inner and outer uncertainty

bounds as black dashed and dotted curves. The uncertainty bounds sit almost on top of the measured curves at this scale. Full values are in Table A-1.

NACA 0012. At 0 deg the distribution follows the expected symmetric shape, with C_p of 0.25 at the first tap (x/c = 0.02) and slightly negative values (about -0.03 to -0.16) over most of the rest of the chord. As the angle of attack increases the C_p on the upper surface grows more negative (this pulls velocity faster because Bernoulli's equation) and the lower surface moves toward positive pressure as expected. The first tap reaches Cp = -2.06 at 8 deg and -2.68 at 10 deg, and the highest value of C_p sits at the first tap at both angles. At 12 deg the distribution changes character: the first tap drops back to C_p = -0.76 and the upper surface stays almost flat between about -0.6 and -0.8 across the chord. This is characteristic of a separated flow, which means the airfoil stalled between 10 and 12 deg. The stagnation values on the lower surface (the upper surface data taken at negative angles) reached Cp = 1.12 at -8 deg and 1.26 at -10 deg. A pressure coefficient cannot exceed 1 in incompressible flow, so these values point to a bias in the reference pressures of roughly 10 to 25 percent at those points. That is much larger than the instrument uncertainty, and possible causes are a difference between the dynamic pressure measured at the pitot tube and the flow at the model, and blockage at high angle of attack.

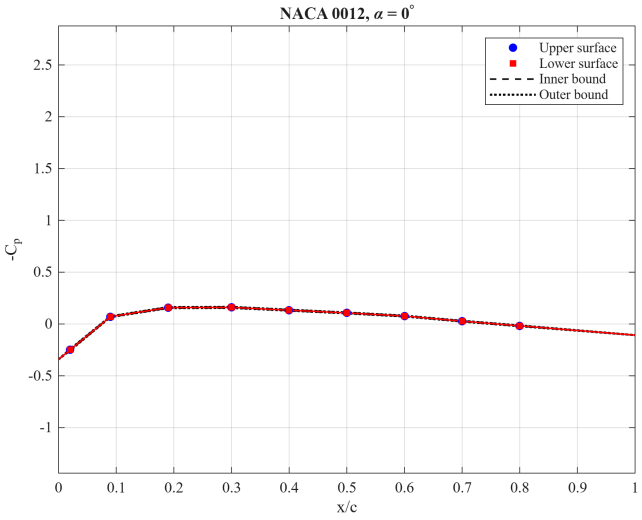


Fig. 2 NACA 0012 -C_p distribution at α = 0 deg (upper surface, blue; lower surface, red; inner and outer uncertainty bounds, black). The two surfaces are identical, so no lift is produced.

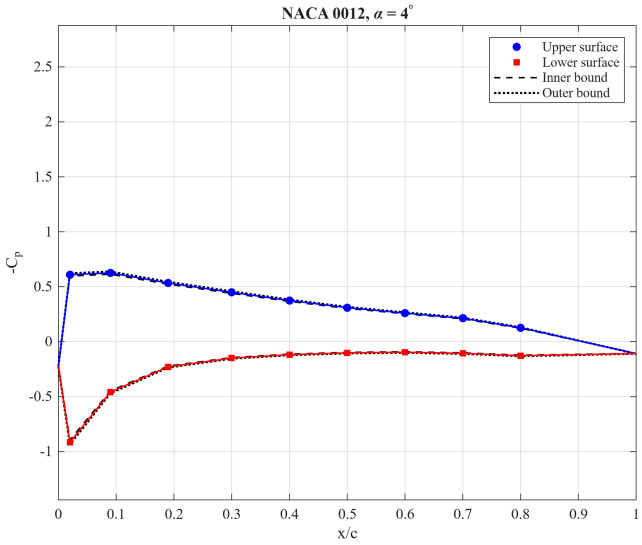


Fig. 3 NACA 0012 -C_p distribution at α = 4 deg.

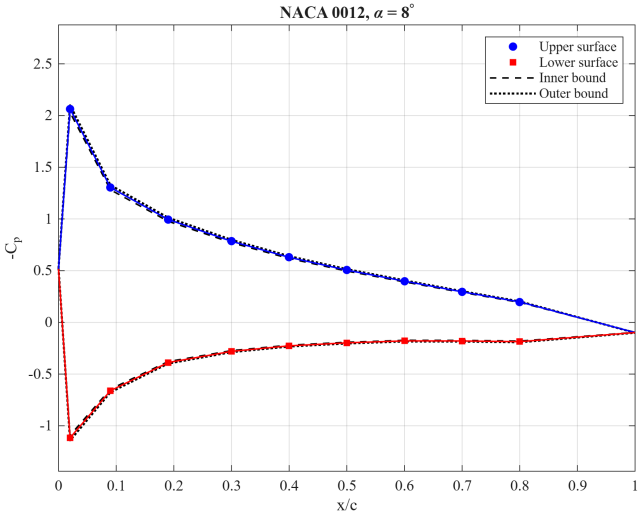


Fig. 4 NACA 0012 -C_p distribution at α = 8 deg.

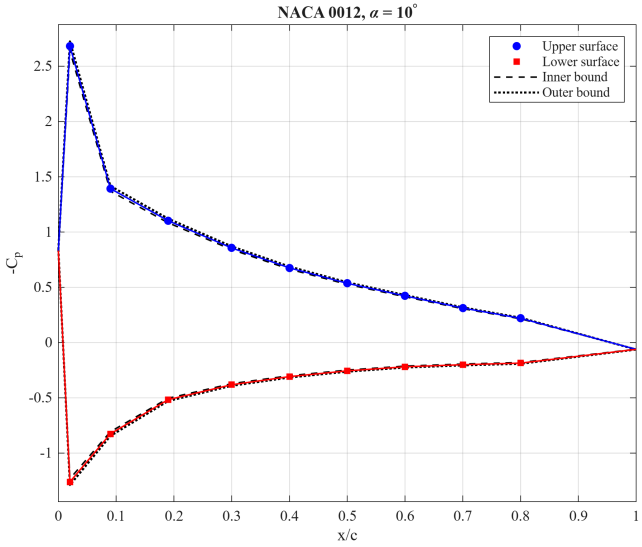


Fig. 5 NACA 0012 -C_p distribution at α = 10 deg.

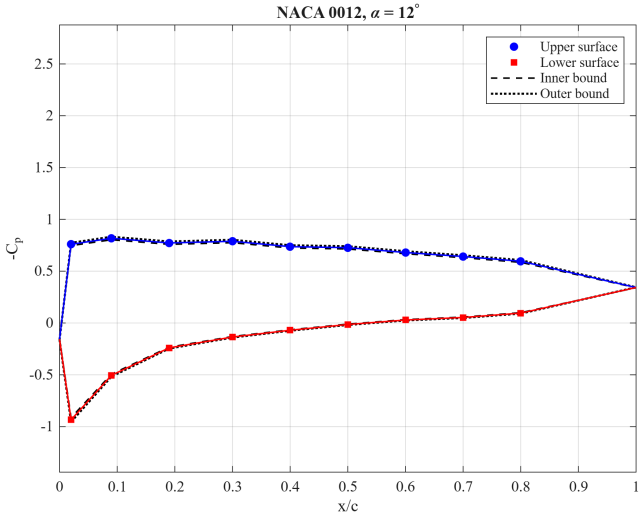


Fig. 6 NACA 0012 -C_p distribution at α = 12 deg.

NACA 4412. At 0 deg the upper surface shows C_p of 0.92 at the leading edge tap and then a -C_p of about -0.6 to -0.7 over the front half of the upper surface, until an anomaly at x/c = 0.61, which we will get into more later on. Overall though we can see a solid difference of pressure coefficients between the upper and lower surface in the NACA4412 airfoil at $\alpha = 0$, meaning that this airfoil is able to produce lift at zero angle of attack, this can therefore be deduced as caused by the different camber (the only difference between the two airfoils tested). Moreover, as the angle of attack increases, the maximum observed pressure coefficient on the upper and lower surfaces continues to grow, and ends up at or ahead of x/c = 0.05, with no tap between leading edge tap (x/c = 0) and x/c = 0.05 to resole it, the upper surface reaches C_p = -1.8 at x/c = 0.05 at 8 deg and -2.4 at 12 deg. The lower surface stays between about -0.3 and +0.4 and shifts toward positive pressure with angle of attack, we note a major shift in this behavior at 14 degrees angle of attack, which indicates separation and stall, so our critical angle of attack lies between 12 and 14 degrees.

As noted in the former paragraph, one feature does not follow the expected shape. That is, at every angle of attack the upper surface tap at x/c = 0.61 (port 7) reads a Cp of about +0.05 to +0.08, while its neighbors at x/c = 0.40 and 0.81 read between -0.7 and -0.2. The reading is very close to the port 0 reference at every angle, which produces the sharp dip toward zero at x/c of about 0.6 in the -Cp plots. Logically, a real pressure distribution cannot do this, and the lower surface shows nothing similar. The most likely cause is that the scannivalve selector knob was not seated exactly on port 7 when its data

were recorded, as a lab teaching assistant suggested, so the reading reflects something close to the reference pressure instead of the tap. The data could not be retaken. The main results in this report keep this tap as measured, and Sec. III.B shows how much the force coefficients change when it is removed.

At 14 deg the readings from taps 1 and 2 on the upper surface and tap 1 on the lower surface were at the scannivalve limit of -10.62 in. H2O. Those taps were excluded, which removes the peak C_p value and makes the 14 deg force coefficients unreliable. That angle is listed in the tables but left off the lift and drag figures.

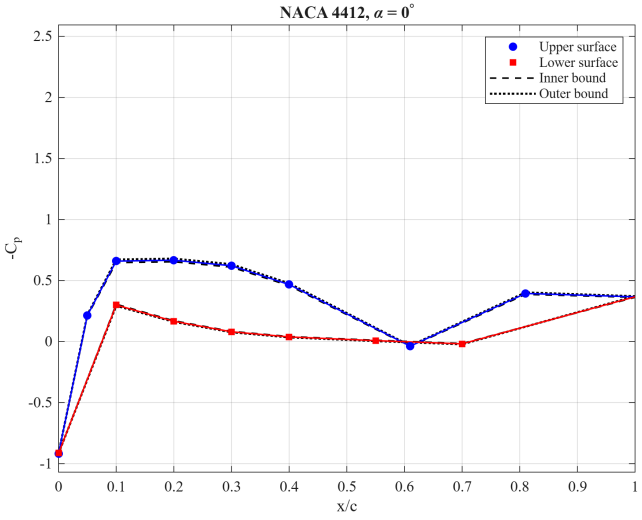


Fig. 7 NACA 4412 -C_p distribution at α = 0 deg.

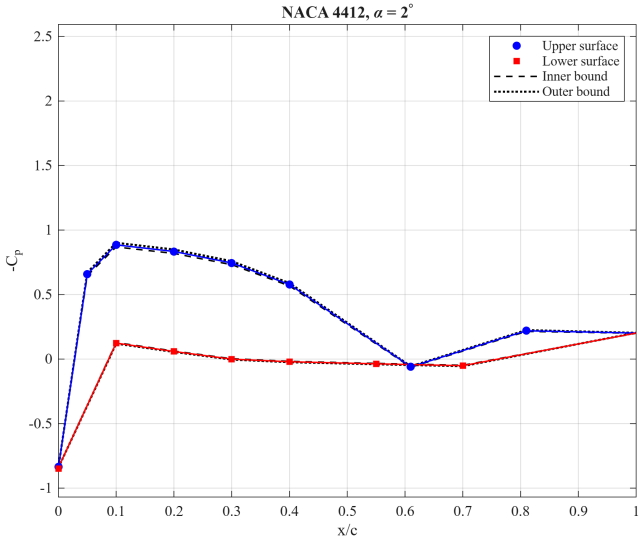


Fig. 8 NACA 4412 -C_p distribution at α = 2 deg.

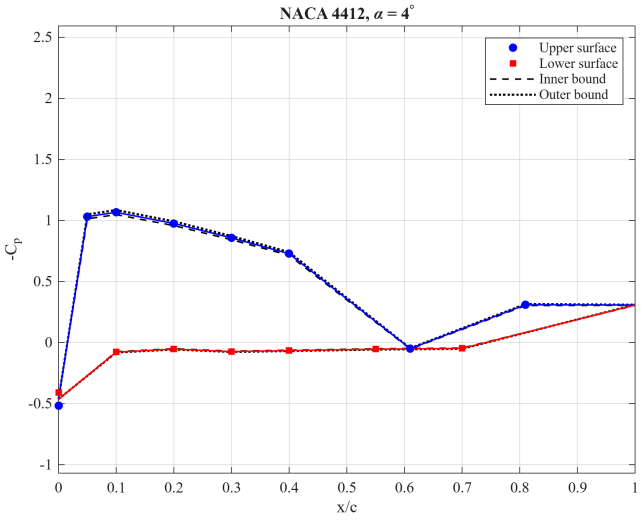


Fig. 9 NACA 4412 -C_p distribution at α = 4 deg.

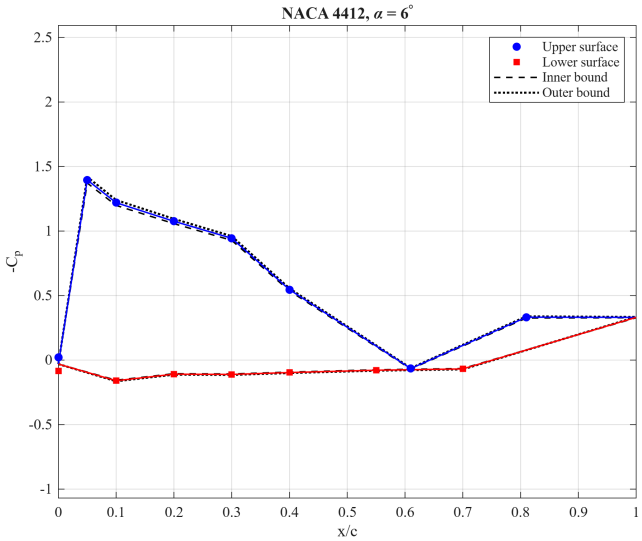


Fig. 10 NACA 4412 -C_p distribution at α = 6 deg.

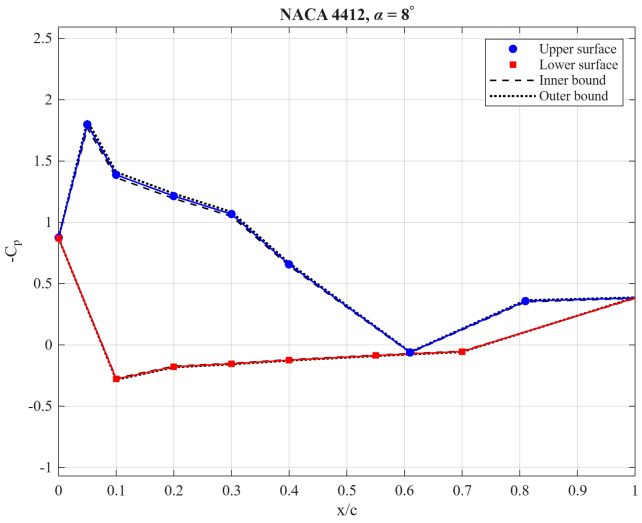


Fig. 11 NACA 4412 -C_p distribution at α = 8 deg.

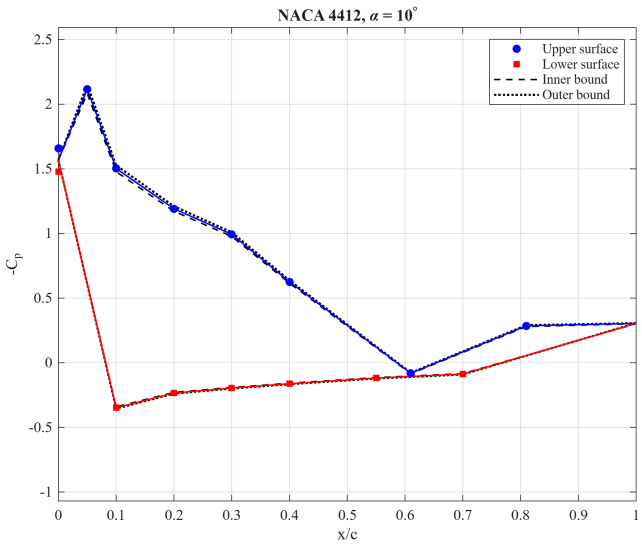


Fig. 12 NACA 4412 -C_p distribution at α = 10 deg.

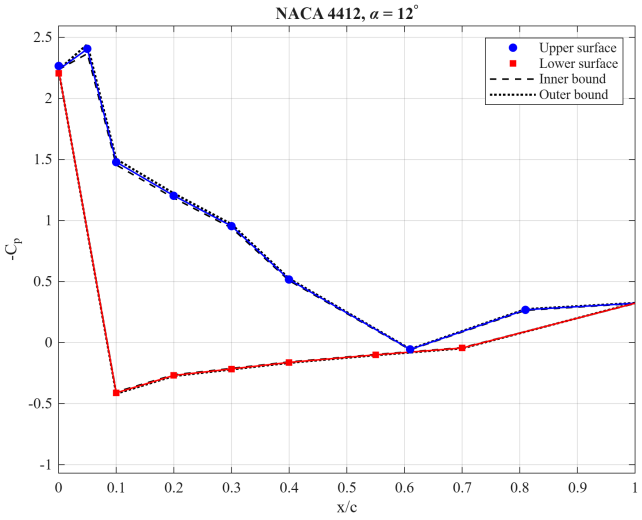


Fig. 13 NACA 4412 -C_p distribution at α = 12 deg.

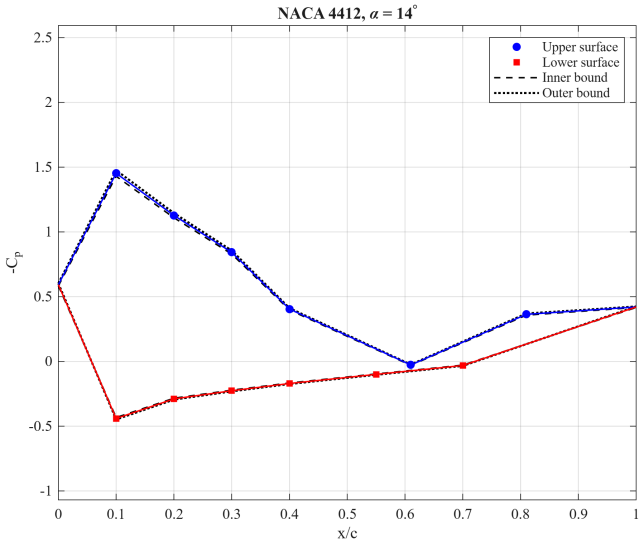


Fig. 14 NACA 4412 -C_p distribution at α = 14 deg.

B. Lift, Drag, and Lift-to-Drag Curves

Figures 15 and 16 show C_l, C_d, and C_l/C_d against angle of attack for each airfoil, with error bars from the inner and outer bound method and the thin airfoil theory lift line ($C_l = 2\pi(\alpha - \alpha_{L0})$) for comparison. The NACA 0012 figure only shows α of 0 deg and above, because its negative angle results are mirror images of the positive ones by construction (the lower surface at +α is the upper surface at -α). The 14 deg point of the NACA 4412 is left off for the reason given in Sec. III.A. Values for every angle are in Table A-2.

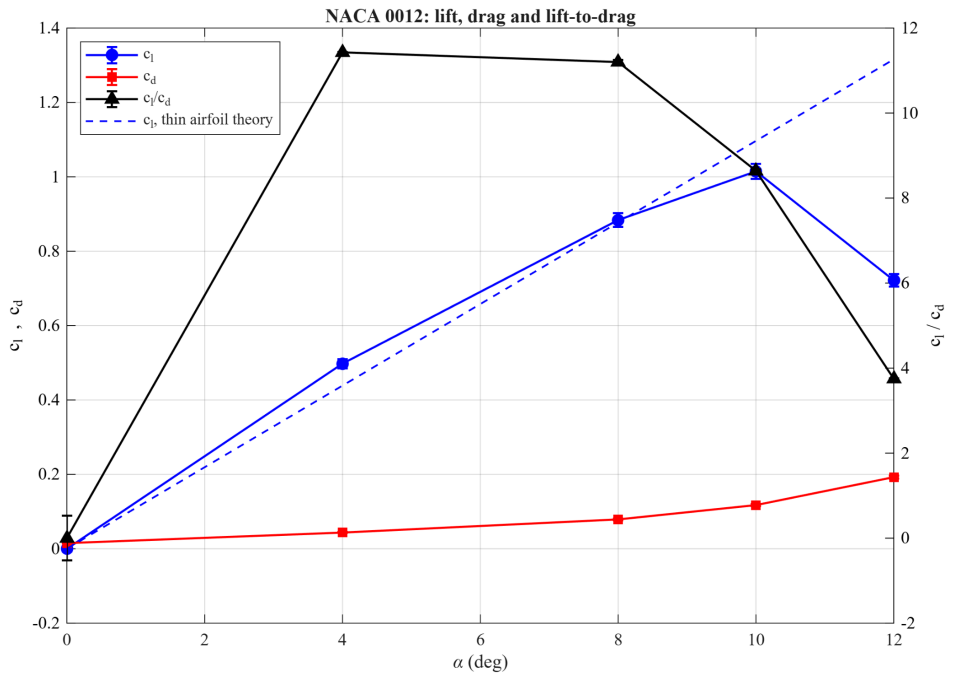


Fig. 15 NACA 0012 lift, drag, and lift-to-drag ratio versus angle of attack with uncertainty bars and thin airfoil theory lift line.

NACA 0012. The lift curve is linear from about -8 to 8 deg, with a fitted slope of 0.1133 per deg (6.49 per rad) and a zero-lift angle of 0.00 deg. The thin airfoil value is $2\pi = 6.28$ per rad, so the measured slope is about 3 percent higher than theory. Lift increases from $C_l = 0.498$ at 4 deg to 0.884 at 8 deg and reaches a maximum of 1.015 at 10 deg. At 12 deg it drops to 0.722, which is consistent with the flat, separated upper surface pressure distribution seen in Fig. 6. The drag coefficient from pressure integration is 0.015 at 0 deg and rises to 0.044 at 4 deg, 0.079 at 8 deg, 0.117 at 10 deg, and 0.193 at 12 deg. The lift to drag ratio reaches its maximum of 11.4 at 4 deg (compared to 11.2 at 8 deg) and falls to 8.6 at 10 deg and 3.7 at 12 deg. It's important to distinguish that this is not the same angle as maximum lift (which occurs at 10 deg). Because drag grows faster than lift as the angle of attack increases, so C_l/C_d reaches its peak well before C_l does.

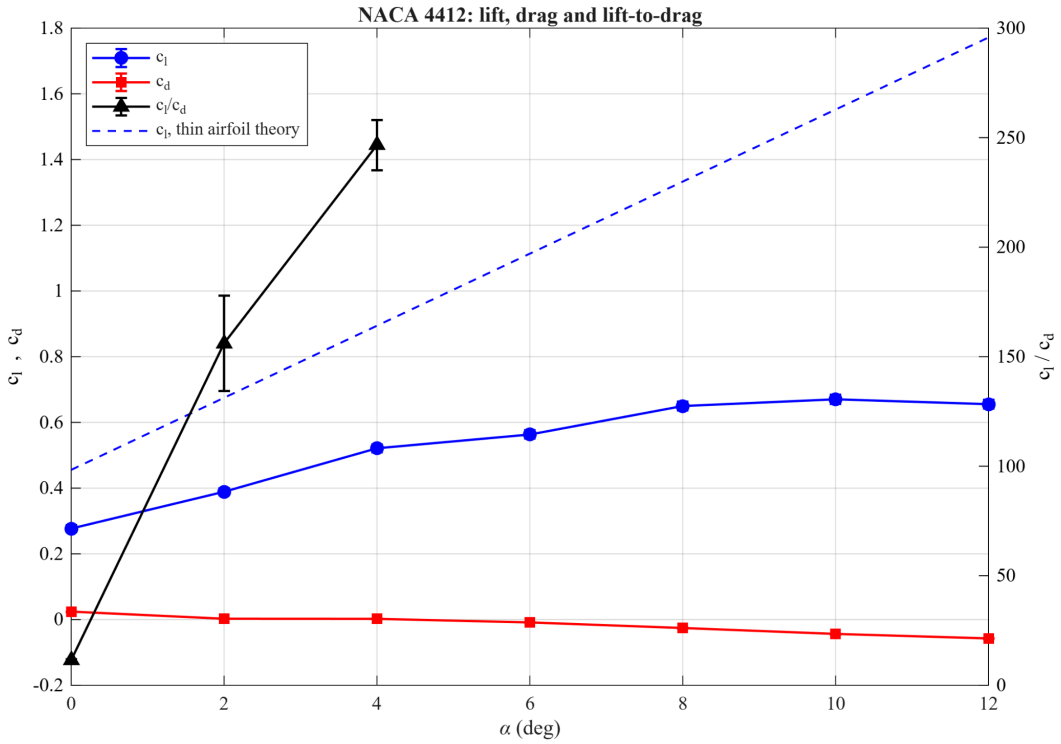


Fig. 16 NACA 4412 lift, drag, and lift-to-drag ratio versus angle of attack with uncertainty bars and thin airfoil theory lift line (all taps kept, 14 deg omitted).

NACA 4412. With all taps as measured, C_l is 0.277 at 0 deg, 0.521 at 4 deg, 0.650 at 8 deg, and reaches a maximum of 0.670 at 10 deg. The fitted slope from 0 to 8 deg is 0.0460 per deg (2.64 per rad), and the extrapolated zero lift angle is -6.4 deg. Thin airfoil theory for a NACA 4412 camber line gives a zero lift angle of -4.15 deg and C_l = 0.455 at 0 deg. The drag coefficient is 0.024 at 0 deg, falls to 0.002 at 2 and 4 deg, and is negative from 6 to 12 deg (-0.009 to -0.057). A negative C_d is not possible, so the C_l/C_d values for this airfoil (for example 246.6 at 4 deg) come from a near zero drag and are not reported as results.

Table 1 shows how the NACA 4412 results change when the anomalous port 7 tap (x/c = 0.61) is removed from the upper surface.

Table 1 NACA 4412 lift and drag coefficients with and without the port 7 tap.

α (deg)	c_l, tap kept	c_l, tap removed	c_d, tap kept	c_d, tap removed
0	0.277	0.385	0.0242	0.0309
4	0.521	0.650	0.0021	0.0207
8	0.650	0.778	-0.0258	0.0013
10	0.670	0.790	-0.0437	-0.0137
12	0.656	0.755	-0.0574	-0.0290

Removing the tap raises C_l by about 0.10 to 0.13 at every angle, and it moves C_d upward by 0.007 to 0.03. It also changes the trailing edge C_p used to close the curve from -0.38 to -0.12, because the linear extrapolation to x/c = 1 used the bad tap. But even then, the 4412 lift curve stays too shallow. The slope only improves to 0.0485 per deg (2.78 per rad), the zero lift angle moves to -8.3 deg, and C_l at 0 deg (0.385) is still below the 0.455 predicted by theory. The maximum C_l of 0.790 still occurs at 10 deg, and C_d is still negative at 10 and 12 deg. Removing the tap improves the results, but it does not explain the slope deficit.

Compared with the NACA 0012, the cambered airfoil produces more lift at the same angle of attack (for example 0.650 versus 0.498 at 4 deg with the tap removed), which is the expected effect of camber. However, the 4412 slope is well below the 0012 slope, where theory predicts the same slope for both, and its maximum lift is lower than that of the symmetric

airfoil, where a cambered airfoil should normally reach a higher maximum. The 4412 C_l also flattens between 10 and 12 deg, which suggests it is close to stall, but the 14 deg data cannot confirm it.

C. Uncertainty

The propagated uncertainty in C_p is about 0.02 for the larger pressure coefficients. For example, the sample point in Appendix B (NACA 4412 upper surface, 8 deg, x/c = 0.10) is Cp = -1.388 with δCp = 0.023, or 1.7 percent. Almost all of this comes from the 13.4 Pa dynamic pressure uncertainty, since the 2.5 Pa scannivalve uncertainty is small next to a dynamic pressure of about 800 Pa. Because that dynamic pressure error is shared by every tap at a given angle, the inner and outer bounds move all taps together, so they are a conservative upper limit on the spread and not an independent random scatter.

Carried through the integration, the uncertainty in lift is δC_l = 0.008 to 0.021 for the NACA 0012 (0.0125 at 4 deg, 0.018 at 8 deg, 0.021 at 10 deg), which is about 2 to 2.5 percent of C_l. For the NACA 4412 it is 0.010 to 0.015, or about 2 to 4 percent. The drag uncertainty is 0.001 to 0.004 for the NACA 0012, again about 2.5 percent, and even smaller in absolute terms for the NACA 4412. At the scale of Figs. 15 and 16 these error bars are barely visible.

These uncertainties only account for the instruments. They do not include the effect of the tap spacing and the leading edge and trailing edge extrapolation, the suspect port 7 reading, or the accuracy of the angle of attack setting. The port 7 tap alone changed the NACA 4412 lift coefficient by 0.10 to 0.13, which is about ten times the instrument uncertainty. The gap between the measured NACA 4412 lift slope and thin airfoil theory is also far larger than the error bars. The differences between the measurements and the expected values in Sec. IV are therefore systematic effects, not scatter.

IV. Conclusion

The NACA 0012 lift curve was realistic, and the NACA 4412 lift curve was not. For the symmetric airfoil the measured lift slope was 6.49 per rad against the thin airfoil value of 6.28, the zero lift angle was 0 deg, and the lift peaked at C_l = 1.015 at 10 deg before dropping at 12 deg, where the upper surface pressures went flat as the flow separated. That is the shape and stall behavior expected for a NACA 0012 at a chord Reynolds number of about $2 * 10^5$, where published data show a lower maximum lift and earlier stall than at higher Reynolds numbers [2, 5]. For the cambered airfoil, the measured slope was only 2.6 to 2.8 per rad (about 42 to 44 percent of theory), and the zero lift angle came out between -6.4 and -8.3 deg against -4.15 deg from thin airfoil theory. Its maximum C_l of 0.67 to 0.79 at 10 deg was below the 0012 value, which is the opposite of what camber normally does to airfoils [2]. What the 4412 did show correctly was the effect of camber on the lift offset: it produced more lift than the 0012 at the same angle of attack and had a negative zero-lift angle.

The lift to drag ratio of the NACA 0012 peaked at 4 deg (11.4), not at the angle of maximum lift (10 deg). Drag rises faster than lift as the angle increases, so the best C_l/C_d occurs at a low angle of attack and the maximum C_l occurs close to stall. For the NACA 4412 the maximum could not be found reliably, because pressure-only drag came out near zero or negative from 6 to 12 deg. It appears to occur at a low angle, well before maximum lift. The drag coefficients for both airfoils are only pressure drag and do not include skin friction. The NACA 0012 values (0.044 at 4 deg, 0.079 at 8 deg) are higher than expected below stall, and the 4412 values fall with angle of attack over the first few degrees, which is not physical. Both of these point to errors in the leading edge region of the integration.

Several things could explain the errors. The first tap is at x/c = 0.02 on the 0012, and on the 4412 the taps at x/c = 0 and 0.05 leave a gap, but the overall peak of the pressure coefficients, and the steepest part of the surface are within those first few percent of the chord, so the extrapolated leading edge Cp controls a large share of the axial force and the lift. The 4412 upper surface tap at x/c = 0.61 read close to the reference pressure at every angle, most likely from the scannivalve knob sitting between positions. It changed C_l by 0.10 to 0.13 and also corrupted the trailing edge extrapolation. Three taps at 14 deg reached the scannivalve limit. Other sources include human error in setting the angle of attack and the scannivalve knob, a possible temperature drift while each angle was scanned, saving the pressure data before the flow was settled, maybe even a blockage at high angle of attack (the test section velocity changed by about 10 to 15 percent at a fixed fan setting), the very high stagnation C_p values (1.12 and 1.26) on the 0012 at negative angles, and the low Reynolds number. The instrument uncertainty was only 2 to 4 percent of C_l and does not include any of these effects. The measurements would possibly improve with more taps close to the leading edge (especially on the upper surface), a check of every scannivalve port before the run, repeated measurements at each angle, and a force balance to measure drag directly.

These results matter because they show what the pressure integration method can and cannot do. With a symmetric airfoil and reasonable tap coverage, the lift slope and stall can be recovered from surface pressures, and here the 0012 slope came within about 3 percent of theory. Drag from pressure alone is very sensitive to the leading edge and cannot be relied on. The

4412 results also show that one bad port or a small gap in tap coverage can move a force coefficient by much more than the instrument uncertainty. Any use of these coefficients should rely on the NACA 0012 lift curve and treat the drag and 4412 values as qualitative.

Appendix A: Tabular Results

Table A-1 contains the pressure coefficients and their propagated uncertainties for every tap and angle of attack in the main run (all taps kept). The value in each cell is C_p, and the number below it is the uncertainty δC_p. Taps marked "sat." reached the scannivalve limit of -10.624 in. H_2O and were excluded. Table A-2 contains the resulting lift and drag coefficients. Table A-3 lists the scannivalve port to tap map.

Table A-1a NACA 0012 upper surface C_p with δC_p below each value. The lower surface at α equals the upper surface at -α.

x/c	-12	-10	-8	-4	0	4	8	10	12
0.02	0.934 ±0.020	1.260 ±0.029	1.116 ±0.025	0.915 ±0.020	0.250 ±0.006	-0.609 ±0.012	-2.064 ±0.036	-2.681 ±0.045	-0.760 ±0.014
0.09	0.505 ±0.012	0.826 ±0.019	0.661 ±0.015	0.459 ±0.011	-0.071 ±0.004	-0.626 ±0.012	-1.304 ±0.023	-1.393 ±0.024	-0.818 ±0.015
0.19	0.240 ±0.007	0.516 ±0.012	0.389 ±0.010	0.228 ±0.006	-0.158 ±0.005	-0.534 ±0.011	-0.995 ±0.017	-1.103 ±0.019	-0.772 ±0.014
0.3	0.135 ±0.005	0.381 ±0.010	0.280 ±0.007	0.149 ±0.005	-0.160 ±0.005	-0.449 ±0.009	-0.784 ±0.014	-0.856 ±0.015	-0.789 ±0.014
0.4	0.070 ±0.004	0.308 ±0.008	0.227 ±0.007	0.118 ±0.005	-0.133 ±0.005	-0.375 ±0.008	-0.629 ±0.011	-0.676 ±0.012	-0.737 ±0.013
0.5	0.015 ±0.004	0.256 ±0.007	0.198 ±0.006	0.102 ±0.005	-0.107 ±0.004	-0.309 ±0.007	-0.505 ±0.009	-0.536 ±0.010	-0.727 ±0.013
0.6	-0.029 ±0.004	0.219 ±0.007	0.179 ±0.006	0.096 ±0.005	-0.077 ±0.004	-0.260 ±0.006	-0.399 ±0.008	-0.422 ±0.008	-0.680 ±0.012
0.7	-0.053 ±0.004	0.200 ±0.006	0.180 ±0.006	0.106 ±0.005	-0.027 ±0.004	-0.213 ±0.005	-0.295 ±0.006	-0.313 ±0.006	-0.641 ±0.012
0.8	-0.095 ±0.005	0.186 ±0.006	0.187 ±0.006	0.127 ±0.005	0.018 ±0.004	-0.126 ±0.004	-0.198 ±0.005	-0.220 ±0.005	-0.596 ±0.011

Table A-1b NACA 4412 upper surface C_p with δC_p below each value. The tap at x/c = 0.61 is the suspect port 7 reading.

x/c	0	2	4	6	8	10	12	14
0	0.919 ±0.018	0.835 ±0.015	0.518 ±0.010	-0.022 ±0.003	-0.875 ±0.015	-1.658 ±0.027	-2.265 ±0.036	sat.
0.05	-0.214 ±0.005	-0.659 ±0.012	-1.032 ±0.018	-1.396 ±0.024	-1.798 ±0.030	-2.117 ±0.034	-2.405 ±0.039	sat.
0.1	-0.659 ±0.013	-0.885 ±0.016	-1.067 ±0.019	-1.220 ±0.021	-1.388 ±0.023	-1.502 ±0.025	-1.479 ±0.024	-1.453 ±0.023
0.2	-0.666 ±0.013	-0.834 ±0.015	-0.976 ±0.017	-1.076 ±0.019	-1.214 ±0.020	-1.191 ±0.020	-1.203 ±0.019	-1.128 ±0.018
0.3	-0.621 ±0.012	-0.745 ±0.014	-0.858 ±0.015	-0.944 ±0.016	-1.067 ±0.018	-0.994 ±0.016	-0.955 ±0.016	-0.843 ±0.014
0.4	-0.467 ±0.009	-0.578 ±0.011	-0.728 ±0.013	-0.546 ±0.010	-0.656 ±0.011	-0.626 ±0.011	-0.516 ±0.009	-0.404 ±0.007
0.61	0.037 ±0.004	0.059 ±0.004	0.049 ±0.003	0.066 ±0.003	0.061 ±0.003	0.081 ±0.003	0.056 ±0.003	0.027 ±0.003
0.81	-0.394 ±0.008	-0.219 ±0.005	-0.310 ±0.006	-0.332 ±0.006	-0.358 ±0.007	-0.285 ±0.006	-0.269 ±0.005	-0.364 ±0.006

Table A-1c NACA 4412 lower surface C_p with δC_p below each value.

x/c	0	2	4	6	8	10	12	14
0	0.913 ±0.017	0.849 ±0.016	0.409 ±0.008	0.083 ±0.003	-0.869 ±0.015	-1.477 ±0.024	-2.206 ±0.035	sat.
0.1	-0.300 ±0.007	-0.124 ±0.004	0.078 ±0.004	0.160 ±0.004	0.278 ±0.006	0.348 ±0.006	0.411 ±0.007	0.441 ±0.008
0.2	-0.166 ±0.005	-0.059 ±0.004	0.053 ±0.003	0.109 ±0.004	0.178 ±0.004	0.234 ±0.005	0.269 ±0.005	0.289 ±0.005
0.3	-0.080 ±0.004	0.000 ±0.003	0.073 ±0.003	0.111 ±0.004	0.154 ±0.004	0.195 ±0.004	0.216 ±0.005	0.226 ±0.005
0.4	-0.038 ±0.004	0.021 ±0.003	0.066 ±0.003	0.097 ±0.004	0.124 ±0.004	0.162 ±0.004	0.162 ±0.004	0.170 ±0.004

x/c	0	2	4	6	8	10	12	14
0.55	-0.008 ±0.003	0.037 ±0.003	0.054 ±0.003	0.079 ±0.003	0.087 ±0.003	0.118 ±0.004	0.100 ±0.003	0.100 ±0.003
0.7	0.018 ±0.004	0.051 ±0.003	0.048 ±0.003	0.068 ±0.003	0.054 ±0.003	0.087 ±0.003	0.044 ±0.003	0.031 ±0.003

Table A-2a NACA 0012 lift and drag coefficients with uncertainties (inner and outer bound method).

α (deg)	c_l	δc_l	c_d	δc_d	c_l/c_d	$\delta(c_l/c_d)$
-12	-0.7216	0.0164	0.1926	0.0040	-3.75	0.01
-10	-1.0147	0.0206	0.1174	0.0029	-8.64	0.04
-8	-0.8838	0.0184	0.0789	0.0020	-11.20	0.05
-4	-0.4977	0.0125	0.0436	0.0011	-11.43	0.01
0	0.0000	0.0079	0.0151	0.0000	0.00	0.52
4	0.4977	0.0125	0.0436	0.0011	11.43	0.01
8	0.8838	0.0184	0.0789	0.0020	11.20	0.05
10	1.0147	0.0206	0.1174	0.0029	8.64	0.04
12	0.7216	0.0164	0.1926	0.0040	3.75	0.01

Table A-2b NACA 4412 lift and drag coefficients with uncertainties (inner and outer bound method).

α (deg)	c_l	δc_l	c_d	δc_d	c_l/c_d	$\delta(c_l/c_d)$
0	0.2769	0.0105	0.0242	0.0001	11.46	0.49
2	0.3889	0.0109	0.0025	0.0003	156.09	21.80
4	0.5213	0.0121	0.0021	0.0000	246.63	11.50
6	0.5631	0.0125	-0.0088	0.0002	-64.25	2.80
8	0.6498	0.0137	-0.0258	0.0006	-25.22	1.08
10	0.6703	0.0138	-0.0437	0.0008	-15.33	0.59
12	0.6555	0.0136	-0.0574	0.0011	-11.41	0.46
14	0.5890	0.0117	0.0605	0.0016	9.73	0.06

The 4412 c_l/c_d values at 2 and 4 deg come from drag coefficients near zero, and the values from 6 to 12 deg come from negative drag, so none of them are physically meaningful. The 14 deg row used only 12 of the 15 taps (three taps saturated) and is not reliable.

Table A-3 Scannivalve port to tap map (chordwise position x/c) from the airfoil geometry handout [3].

Port	NACA 0012 x/c	NACA 4412 upper x/c	NACA 4412 lower x/c
1	0.02	0	0
2	0.09	0.05	0.1
3	0.19	0.1	0.2
4	0.30	0.2	0.3
5	0.40	0.3	0.4
6	0.50	0.4	0.55
7	0.60	0.61	0.7
8	0.70	0.81	-
9	0.80	-	-

The 4412 lower surface file also contained an extra port 8 reading at 8 deg that has no matching tap, and it was not used.

Appendix B: Sample Calculations

The following sample calculation uses the NACA 4412 upper surface tap 3 (x/c = 0.10) at α = 8 deg from the main run with all taps kept. The same procedure was used for every tap and angle of attack.

1. Pressure Coefficient

The scannivalve readings at this angle were r_0 = -1.810 in. H_2O (port 0) and r_3 = -6.317 in. H_2O (port 3), and the mean pitot dynamic pressure over the nine port readings was q_∞ = 809.08 Pa. The pressure difference was

$$\Delta p = (r_3 - r_0)(249.09) = (-6.317 - (-1.810))(249.09) = -1122.6 \text{ Pa}$$

and the pressure coefficient was

$$C_p = \Delta p / q_\infty = -1122.6 / 809.08 = -1.388$$

2. Pressure Coefficient Uncertainty

The chart uncertainties of 0.005 V (scannivalve) and 0.01 V (transducer) were converted with the Lab 1 slopes of 504 Pa/V and 1339 Pa/V,

$$\sigma_p = (504)(0.005) = 2.52 \text{ Pa}, \quad \sigma_q = (1339)(0.01) = 13.39 \text{ Pa}$$

so that

$$\delta C_p = \sqrt{[(\sigma_p/q_\infty)^2 + (C_p\sigma_q/q_\infty)^2]} = \sqrt{[(2.52/809.08)^2 + ((-1.388)(13.39)/809.08)^2]} = \sqrt{[(0.00311)^2 + (0.02296)^2]} = 0.0232$$

The outer bound lowers the upper surface C_p to -1.388 - 0.023 = -1.411 (and raises the lower surface C_p by the same amount at each tap), and the inner bound does the reverse, giving -1.365 at this tap.

3. Force Coefficients and Their Uncertainty

Integrating the true distribution at 8 deg with the trapezoidal rule, after closing the curve by linear extrapolation to x/c = 0 and 1, gave $c_n = 0.6399$ and $c_a = -0.1160$. Then

$$c_l = c_n \cos\alpha - c_a \sin\alpha = (0.6399)(0.9903) - (-0.1160)(0.1392) = 0.6498$$

$$c_d = c_n \sin\alpha + c_a \cos\alpha = (0.6399)(0.1392) + (-0.1160)(0.9903) = -0.0258$$

The inner and outer distributions gave $c_l = 0.6361$ and 0.6636 and $c_d = -0.02633$ and -0.02521, so

$$\delta c_l = (0.6636 - 0.6361)/2 = 0.0137, \quad \delta c_d = (-0.02521 - (-0.02633))/2 = 0.00056$$

The lift-to-drag ratio was not evaluated at this point because c_d is negative.

Appendix C: Code

```
%% AEE 360 Lab 2 - Lift and drag from Cp measurements
% NACA 0012 and NACA 4412, 40 Hz tunnel setting.
% I was kinda confused on how to do uncertainty but I'm doing it the way the
% whiteboard suggest: Cp +/- dCp gives an inner and an
% outer bound, cl and cd get integrated for true/inner/outer, and
% d(cl) = (max - min)/2 across those three.
clear; clc; close all;
set(groot, 'defaultAxesFontName','Times New Roman', ...
'defaultTextFontName','Times New Roman', ...
'defaultLegendFontName','Times New Roman');
%matlab 2026 was being stupid so i put these functions in to make my graphs
%look pretty ^^^
%% SECTION 0: constants and settings
inH2O_to_Pa = 249.09;
R_air       = 287.05;
chord       = 0.100;            % m
DPI         = 300;
satLimit = -10.6;               % scannivalve pegs at about -10.624 in H2O
% the machine error chart uncertainties are in VOLTS, so they get converted to
% pressure with the Lab 1 calibration slopes.
% The slopes below are rough refits from my Lab 1 Table A-1
m_trans  = 1339;                % Pa/V, pressure transducer (Lab 1)
m_scanni = 504;                 % Pa/V, scannivalve (Lab 1)
sigma_V_trans  = 0.01;          % V, from the machine error chart
sigma_V_scanni = 0.005;         % V, from the machine error chart
sigma_q      = m_trans*sigma_V_trans;                % Pa (about 13.4)
sigma_scanni = m_scanni*sigma_V_scanni/inH2O_to_Pa;   % in H2O, data column is in H2O (about 0.0101)
useRootTwo   = false;           % false = dP_scan uncertainty used as one term (like the board)
% true  = tap and port 0 both count, so multiply by sqrt(2)
% Port 7 on the 4412 upper surface (x/c = 0.61) reads about the static pressure
% at every angle. TA said just explain it in the report, so i decided to to keep it for the code but
% im gonna once with true to see how much it changes cl and explain that in
% the report.
EXCLUDE_BAD_TAP = false;
extrapMethod = 'linear';        % how to reach x/c = 0 and 1: 'linear' or 'nearest'
fitRange12   = [-8 8];          % "linear region" for the 0012 lift slope (deg)
fitRange44   = [0 8];           % same for the 4412, adjust after looking at the plots
cpAngles12 = [0 4 8 10 12];     % whiteboard says 5 Cp graphs per airfoil
cpAngles44 = [];                % [] = all angles I have. Board list looked like [0 4 8 10 14]
outDir = 'lab2_outputs';
if ~exist(outDir,'dir'), mkdir(outDir); end
if useRootTwo, rt2 = sqrt(2); else, rt2 = 1; end
opts = struct('K',inH2O_to_Pa, 'satLimit',satLimit, ...
```

```
'sig_r',sigma_scanni, 'sig_q',sigma_q, 'rt2',rt2);
%% SECTION 1: airfoil geometry (from the geometry handout)
zt = @(x) (0.12/0.2)*(0.296*sqrt(x) - 0.126*x - 0.3516*x.^2 ...
                    + 0.2843*x.^3 - 0.1015*x.^4);
g12.xt = [2 9 19 30 40 50 60 70 80]/100;
g12.yt =  zt(g12.xt);
g12.xb = g12.xt;
g12.yb = -zt(g12.xt);
g12.yTEu =  zt(1);
g12.yTEl = -zt(1);
g44.xt = [0 .05 .10 .20 .30 .40 .61 .81];
g44.yt = [0 .0449 .0643 .0874 .0975 .0980 .0797 .0458];
g44.xb = [0 .10 .20 .30 .40 .55 .70];
g44.yb = [0 -.0293 -.0274 -.0225 -.0180 -.0119 -.0063];
g44.yTEu = 0;
g44.yTEl = 0;
% my brain hurts
%% SECTION 2: load the data
% Port 0 = test section static (the reference). Ports 1 to N = airfoil taps.
D12  = load_airfoil_file('Tues_600_A_0012.txt',   numel(g12.xt));
D44u = load_airfoil_file('Tues_600_A_4412_U.txt', numel(g44.xt));
D44l = load_airfoil_file('Tues_600_A_4412_L.txt', numel(g44.xb));
if ~isequal(D44u.alpha, D44l.alpha)
   error('4412 upper and lower files do not have the same angles of attack.');
end
if EXCLUDE_BAD_TAP
   D44u.r(:,7) = NaN;           % upper tap 7, x/c = 0.61
   fprintf('*** 4412 upper tap 7 (x/c = 0.61) EXCLUDED ***\n');
end
% remeber what I said earlier in the code abt this mr/mrs ta ^^
fprintf('\n Test conditions \n');
print_conditions('NACA 0012',       D12,  chord, R_air);
print_conditions('NACA 4412 upper', D44u, chord, R_air);
print_conditions('NACA 4412 lower', D44l, chord, R_air);
fprintf('\n Uncertainty inputs \n');
fprintf('sigma_scanni = %.4f in H2O (%.2f Pa),  sigma_q = %.2f Pa\n', ...
       sigma_scanni, sigma_scanni*inH2O_to_Pa, sigma_q);
fprintf('\n Saturated scannivalve readings (dropped from the integration) \n');
report_saturation('NACA 0012',       D12,  satLimit);
report_saturation('NACA 4412 upper', D44u, satLimit);
report_saturation('NACA 4412 lower', D44l, satLimit);
%% SECTION 3: Cp, bounds, cl and cd
res12 = run_case(D12,  [],   g12, opts, extrapMethod);
res44 = run_case(D44u, D44l, g44, opts, extrapMethod);
%% SECTION 4: tables (Appendix A)
TA1_12 = cp_table('NACA 0012', g12, res12);
TA1_44 = cp_table('NACA 4412', g44, res44);
writetable(TA1_12, fullfile(outDir,'TableA1_Cp_0012.csv'));
writetable(TA1_44, fullfile(outDir,'TableA1_Cp_4412.csv'));
% (0012 lower surface Cp = upper surface Cp measured at -alpha)
TA2_12 = coeff_table(res12);
TA2_44 = coeff_table(res44);
writetable(TA2_12, fullfile(outDir,'TableA2_coeffs_0012.csv'));
writetable(TA2_44, fullfile(outDir,'TableA2_coeffs_4412.csv'));
fprintf('\n Table A-2: NACA 0012 \n');  disp(TA2_12);
fprintf('\n Table A-2: NACA 4412 \n');  disp(TA2_44);
fprintf('Table A-1 (Cp and dCp vs x/c) written to csv in %s\n', outDir);
%% SECTION 5: numbers for the discussion
aL0_12 = 0;
aL0_44 = alpha_L0_4digit(0.04, 0.4);     % thin airfoil theory zero-lift angle
fprintf('\n Lift curve and l/d summary \n');
lift_summary('NACA 0012', res12, fitRange12, aL0_12);
lift_summary('NACA 4412', res44, fitRange44, aL0_44);
%% SECTION 6: figures
plot_cp_set(res12, g12, '0012', 'NACA 0012', cpAngles12, outDir, DPI);
plot_cp_set(res44, g44, '4412', 'NACA 4412', cpAngles44, outDir, DPI);
plot_aero(aero_plot_data(res12), '0012', 'NACA 0012', aL0_12, outDir, DPI);
plot_aero(aero_plot_data(res44), '4412', 'NACA 4412', aL0_44, outDir, DPI);
plot_aero_panels(aero_plot_data(res12), '0012', 'NACA 0012', aL0_12, fitRange12, outDir, DPI);
plot_aero_panels(aero_plot_data(res44), '4412', 'NACA 4412', aL0_44, fitRange44, outDir, DPI);
%% SECTION 7: sample calculation for Appendix B (4412 upper, alpha = 8 deg, tap 3)
```

```
kS = find(abs(D44u.alpha - 8) < 1e-6, 1);
jS = 3;
r_tap = D44u.r(kS,jS);  r_0 = D44u.r0(kS);  q_s = D44u.q(kS);
dp_s  = (r_tap - r_0)*inH2O_to_Pa;
fprintf('\n Sample calc: 4412 upper surface, alpha = 8 deg, tap %d (x/c = %.2f) \n', jS, g44.xt(jS));
fprintf('tap reading        = %.3f in H2O\n', r_tap);
fprintf('static (port 0)    = %.3f in H2O\n', r_0);
fprintf('p - p_inf          = (%.3f - (%.3f))(%.2f) = %.2f Pa\n', r_tap, r_0, inH2O_to_Pa, dp_s);
fprintf('q_inf (pitot)      = %.2f Pa\n', q_s);
fprintf('Cp = dp/q          = %.4f\n', dp_s/q_s);
fprintf('dCp                = %.4f  (bounds: %.4f to %.4f)\n', res44.sigCpt(kS,jS), ...
       res44.Cpt(kS,jS) - res44.sigCpt(kS,jS), res44.Cpt(kS,jS) + res44.sigCpt(kS,jS));
d = res44.nom.detail{kS};
fprintf('\nSame angle, integration (extrapolated LE Cp = %.3f, TE Cp = %.3f):\n', d.cpu(1),
d.cpu(end));
fprintf('cn = %.4f, ca = %.4f\n', d.cn, d.ca);
fprintf('cl = cn*cos(a) - ca*sin(a) = %.4f\n', d.cl);
fprintf('cd = cn*sin(a) + ca*cos(a) = %.4f\n', d.cd);
fprintf('cl true / inner / outer = %.4f / %.4f / %.4f  ->  dcl = %.4f\n', ...
       res44.nom.cl(kS), res44.inner.cl(kS), res44.outer.cl(kS), res44.dcl(kS));
fprintf('cd true / inner / outer = %.4f / %.4f / %.4f  ->  dcd = %.4f\n', ...
       res44.nom.cd(kS), res44.inner.cd(kS), res44.outer.cd(kS), res44.dcd(kS));
fprintf('\n booyah baby. Figures and tables are in "%s" folder.\n', outDir);
%%  HELPER FUNCTIONS
function D = load_airfoil_file(fname, nTaps)
% Columns: Hz, T(C), q(Pa), alpha, scanni(inH2O), port, V(m/s), Pamb(kPa)
if ~isfile(fname)
       error('Cannot find %s.', fname);
end
   M = readmatrix(fname, 'FileType','text', 'Delimiter','\t', 'NumHeaderLines',1);
   M(any(isnan(M),2),:) = [];
   alphas = unique(M(:,4))';
   nA = numel(alphas);
   D.alpha = alphas;
   D.r0 = nan(1,nA);
   D.r  = nan(nA,nTaps);
   D.q  = nan(1,nA);  D.T = D.q;  D.V = D.q;  D.Pamb = D.q;
for k = 1:nA
       sub = M(M(:,4) == alphas(k), :);
       p = sub(:,6);
       D.r0(k) = mean(sub(p==0,5));
for j = 1:nTaps                          % ports past nTaps are ignored
if any(p==j), D.r(k,j) = mean(sub(p==j,5)); end
end
       D.q(k) = mean(sub(:,3));    D.T(k) = mean(sub(:,2));
       D.V(k) = mean(sub(:,7));    D.Pamb(k) = mean(sub(:,8));
end
end
function print_conditions(name, D, chord, R)
   T = mean(D.T) + 273.15;
   P = mean(D.Pamb)*1000;
   rho = P/(R*T);
   mu = 1.716e-5*(T/273.15)^1.5*(273.15+110.4)/(T+110.4);   % Sutherland
   V = mean(D.V);
   fprintf('%-16s T = %.1f K, P = %.1f kPa, rho = %.4f kg/m^3, V = %.2f m/s, Re_c = %.3g\n', ...
           name, T, P/1000, rho, V, rho*V*chord/mu);
end
function report_saturation(name, D, satLimit)
   [k,j] = find(D.r <= satLimit);
if isempty(k), fprintf('%s: none\n', name); end
for i = 1:numel(k)
       fprintf('%s: alpha = %g deg, tap %d reads %.3f in H2O\n', name, D.alpha(k(i)), j(i),
D.r(k(i),j(i)));
end
end
function [Cp, sigCp] = cp_from_raw(D, opts)
% Cp = (p_i - p_inf)/q_inf. Tap and port 0 are both gage readings, so the
% difference is p_i - p_inf directly.
   dp = (D.r - D.r0(:))*opts.K;                % Pa
   q  = D.q(:);
   Cp = dp ./ q;
```

```
   Cp(D.r <= opts.satLimit) = NaN;              % throw out saturated taps
% dCp = sqrt( (dCp/dP * dP)^2 + (dCp/dq * dq)^2 ),  dCp/dP = 1/q,  dCp/dq = -Cp/q
   sig_dp = opts.K*opts.rt2*opts.sig_r;
   sigCp  = sqrt((sig_dp./q).^2 + (Cp.*opts.sig_q./q).^2);
end
function res = run_case(Dt, Db, g, opts, method)
% Dt = top surface data, Db = bottom surface data.
% 0012: pass Db = [] and the lower surface at alpha is the upper surface
% at -alpha (symmetric airfoil).
   [Cpt, sigt] = cp_from_raw(Dt, opts);
if isempty(Db)
       Cpb = nan(size(Cpt));  sigb = Cpb;
for k = 1:numel(Dt.alpha)
           m = find(abs(Dt.alpha + Dt.alpha(k)) < 1e-6, 1);
if ~isempty(m)
               Cpb(k,:)  = Cpt(m,:);
               sigb(k,:) = sigt(m,:);
end
end
else
       [Cpb, sigb] = cp_from_raw(Db, opts);
end
   nA = numel(Dt.alpha);
   res.alpha = Dt.alpha;
   res.Cpt = Cpt;  res.Cpb = Cpb;
   res.sigCpt = sigt;  res.sigCpb = sigb;
% nominal, inner bound (smaller loop), outer bound (bigger loop)
%   inner: upper Cp + dCp, lower Cp - dCp
%   outer: upper Cp - dCp, lower Cp + dCp
   names = {'nom','inner','outer'};
   f     = [0, +1, -1];
for s = 1:3
       S.cl = nan(1,nA);  S.cd = S.cl;  S.cn = S.cl;  S.ca = S.cl;  S.nDrop = S.cl;
       S.detail = cell(1,nA);
for k = 1:nA
           d = airfoil_forces(g, Cpt(k,:) + f(s)*sigt(k,:), Cpb(k,:) - f(s)*sigb(k,:), ...
                              Dt.alpha(k), method);
           S.cl(k) = d.cl;  S.cd(k) = d.cd;  S.cn(k) = d.cn;  S.ca(k) = d.ca;
           S.nDrop(k) = d.nDrop;
           S.detail{k} = d;
end
       S.ld = S.cl ./ S.cd;
       res.(names{s}) = S;
end
   res.cl = res.nom.cl;  res.cd = res.nom.cd;  res.ld = res.nom.ld;
   res.nDrop = res.nom.nDrop;
   res.dcl = half_range(res, 'cl');       % (max - min)/2 over true/inner/outer
   res.dcd = half_range(res, 'cd');
   res.dld = half_range(res, 'ld');
end
function h = half_range(res, fld)
   st = [res.nom.(fld); res.inner.(fld); res.outer.(fld)];
   h = (max(st,[],1) - min(st,[],1))/2;
end
function d = airfoil_forces(g, cpu, cpl, alphaDeg, method)
% Integrates the pressure distribution around the airfoil. The taps don't
% reach x/c = 0 or 1, so the curve is extended to LE and TE to close the
% loop, and both surfaces share one Cp at each end so they actually meet.
   d = struct('cl',NaN,'cd',NaN,'cn',NaN,'ca',NaN,'nDrop',0, ...
'xu',[],'yu',[],'cpu',[],'xl',[],'yl',[],'cpl',[]);
   ku = ~isnan(cpu);   kl = ~isnan(cpl);
   d.nDrop = sum(~ku) + sum(~kl);
if nnz(ku) < 2 || nnz(kl) < 2, return; end
   xu = g.xt(ku);  yu = g.yt(ku);  cu = cpu(ku);
   xl = g.xb(kl);  yl = g.yb(kl);  cl_ = cpl(kl);
   cLE = 0.5*(edge_value(xu,cu,0,method) + edge_value(xl,cl_,0,method));
   cTE = 0.5*(edge_value(xu,cu,1,method) + edge_value(xl,cl_,1,method));
   [d.xu, d.yu, d.cpu] = close_surface(xu, yu, cu,  cLE, cTE, g.yTEu);
   [d.xl, d.yl, d.cpl] = close_surface(xl, yl, cl_, cLE, cTE, g.yTEl);
   d.cn = trapz(d.xl, d.cpl) - trapz(d.xu, d.cpu);      % int (Cp_l - Cp_u) dx/c
   d.ca = trapz(d.yu, d.cpu) - trapz(d.yl, d.cpl);      % int Cp_u dy_u - Cp_l dy_l
```

```
   a = alphaDeg*pi/180;
   d.cl = d.cn*cos(a) - d.ca*sin(a);
   d.cd = d.cn*sin(a) + d.ca*cos(a);
end
function c0 = edge_value(x, c, x0, method)
   [dmin, i0] = min(abs(x - x0));
if dmin < 1e-9
       c0 = c(i0);
return
end
if strcmp(method,'nearest')
       c0 = c(i0);
else
       [~, ord] = sort(abs(x - x0));
       a = ord(1);  b = ord(2);
       c0 = c(a) + (c(b) - c(a))/(x(b) - x(a))*(x0 - x(a));
end
end
function [x2, y2, c2] = close_surface(x, y, c, cLE, cTE, yTE)
   k = x > 1e-9 & x < 1-1e-9;
   x2 = [0, x(k), 1];
   y2 = [0, y(k), yTE];
   c2 = [cLE, c(k), cTE];
end
function T = cp_table(name, g, res)
% everything plotted: Cp and dCp at each tap (bounds are Cp -/+ dCp)
   A = [];  S = strings(0,1);  X = [];  C = [];  E = [];
   nt = numel(g.xt);  nb = numel(g.xb);
for k = 1:numel(res.alpha)
       A = [A; repmat(res.alpha(k), nt+nb, 1)];
       S = [S; repmat("upper",nt,1); repmat("lower",nb,1)];
       X = [X; g.xt(:); g.xb(:)];
       C = [C; res.Cpt(k,:)'; res.Cpb(k,:)'];
       E = [E; res.sigCpt(k,:)'; res.sigCpb(k,:)'];
end
   T = table(repmat(string(name),numel(A),1), A, S, X, C, E, ...
'VariableNames', {'Airfoil','alpha_deg','Surface','x_over_c','Cp','dCp'});
end
function T = coeff_table(res)
   T = table(res.alpha', res.cl', res.inner.cl', res.outer.cl', res.dcl', ...
             res.cd', res.inner.cd', res.outer.cd', res.dcd', ...
             res.ld', res.dld', res.nDrop', ...
'VariableNames', {'alpha_deg','cl','cl_inner','cl_outer','dcl', ...
'cd','cd_inner','cd_outer','dcd','l_over_d','d_l_over_d','taps_dropped'});
end
% i hate my life
function a0 = alpha_L0_4digit(m, p)
% thin airfoil theory zero-lift angle (deg) for a NACA 4-digit camber line
   th = linspace(0, pi, 4001);
   x = (1 - cos(th))/2;
   dzdx = zeros(size(x));
   f = x < p;
   dzdx(f)  = 2*m/p^2*(p - x(f));
   dzdx(~f) = 2*m/(1-p)^2*(p - x(~f));
   a0 = -(1/pi)*trapz(th, dzdx.*(cos(th) - 1))*180/pi;
end
function lift_summary(name, res, fitRange, aL0_theory)
   a = res.alpha;  cl = res.cl;
   in = a >= fitRange(1) & a <= fitRange(2) & ~isnan(cl);
   p = polyfit(a(in), cl(in), 1);
   fprintf('\n%s\n', name);
   fprintf('  lift slope over %g to %g deg: %.4f /deg = %.3f /rad  (thin airfoil: %.3f /rad)\n', ...
           fitRange(1), fitRange(2), p(1), p(1)*180/pi, 2*pi);
   fprintf('  zero-lift angle from fit: %.2f deg  (thin airfoil: %.2f deg)\n', -p(2)/p(1),
aL0_theory);
   [clmax, i1] = max(cl);
   fprintf('  max cl in the data: %.3f at alpha = %g deg\n', clmax, a(i1));
   ldv = res.ld;
   ldv(res.cd <= 0) = NaN;                  % l/d means nothing when cd = 0
   [ldmax, i2] = max(ldv);
   fprintf('  max l/d (cd > 0 points only): %.2f at alpha = %g deg\n', ldmax, a(i2));
```

```
if any(res.cd <= 0)
       fprintf('  note: cd = 0 at alpha = %s deg. This is pressure drag only (no skin\n',
num2str(a(res.cd <= 0)));
       fprintf('  friction).\n');
end
end
function plot_cp_set(res, g, tag, name, angles, outDir, DPI)
if isempty(angles), idx = 1:numel(res.alpha); else, idx = find(ismember(res.alpha, angles)); end
if isempty(idx), warning('No matching angles for %s Cp plots.', tag); return; end
% one shared y range so the tiles compare fairly ik ill lose points
% otherwise
   A = res.Cpt(idx,:);  B = res.Cpb(idx,:);
   allv = [-A(:)', -B(:)'];
for k = idx
       dd = res.outer.detail{k};
if ~isempty(dd.cpu), allv = [allv, -dd.cpu, -dd.cpl]; end
end
   yl = [min(allv) - 0.15, max(allv) + 0.15];
   fig = figure('Color','w','Position',[50 50 1300 900]);
   nRows = ceil(numel(idx)/3);
for i = 1:numel(idx)
       ax = subplot(nRows, 3, i);
       draw_cp_axes(ax, g, res, idx(i), yl, i == 1, name);
end
   save_fig(fig, fullfile(outDir, ['Fig_Cp_' tag '_all.png']), DPI);
for k = idx
       f = figure('Color','w','Visible','off','Position',[100 100 650 480]);
       ax = axes(f);
       draw_cp_axes(ax, g, res, k, yl, true, name);
       fname = sprintf('Fig_Cp_%s_a%s.png', tag, strrep(num2str(res.alpha(k)),'-','neg'));
       save_fig(f, fullfile(outDir, fname), DPI);
       close(f);
end
end
function draw_cp_axes(ax, g, res, k, yl, showLegend, name)
% black/blue/red only, line styles tell the curves apart
   hold(ax,'on');
   di = res.inner.detail{k};   dOut = res.outer.detail{k};   dn = res.nom.detail{k};
   hi = []; ho = [];
if ~isempty(di.xu)
       hi = plot(ax, di.xu, -di.cpu, 'k--', 'LineWidth',1);
       plot(ax, di.xl, -di.cpl, 'k--', 'LineWidth',1, 'HandleVisibility','off');
       ho = plot(ax, dOut.xu, -dOut.cpu, 'k:', 'LineWidth',1.4);
       plot(ax, dOut.xl, -dOut.cpl, 'k:', 'LineWidth',1.4, 'HandleVisibility','off');
       plot(ax, dn.xu, -dn.cpu, 'b-', 'LineWidth',1, 'HandleVisibility','off');
       plot(ax, dn.xl, -dn.cpl, 'r-', 'LineWidth',1, 'HandleVisibility','off');
end
   hu = plot(ax, g.xt, -res.Cpt(k,:), 'bo', 'MarkerFaceColor','b', 'MarkerSize',5);
   hl = plot(ax, g.xb, -res.Cpb(k,:), 'rs', 'MarkerFaceColor','r', 'MarkerSize',5);
   grid(ax,'on');  box(ax,'on');
   xlim(ax,[0 1]);  ylim(ax,yl);
   xlabel(ax,'x/c');  ylabel(ax,'-C_p');
   title(ax, sprintf('%s, \\alpha = %g^\\circ', name, res.alpha(k)));
if showLegend
       h = [hu hl hi ho];
       lab = {'Upper surface','Lower surface'};
if ~isempty(hi), lab = [lab, {'Inner bound','Outer bound'}]; end
       legend(ax, h, lab, 'Location','northeast');
end
end
function plot_aero(res, tag, name, aL0, outDir, DPI)
% one graph per airfoil: cl and cd on the left axis, l/d on the right
   a = res.alpha;
   fig = figure('Color','w','Position',[100 100 820 560]);
   ax = axes(fig);  hold(ax,'on');
   yyaxis(ax,'left');
   h1 = errorbar(ax, a, res.cl, res.dcl, 'bo-', 'MarkerFaceColor','b', 'LineWidth',1.2);
   h2 = errorbar(ax, a, res.cd, res.dcd, 'rs-', 'MarkerFaceColor','r', 'LineWidth',1.2);
   xx = linspace(min(a), max(a), 50);
   h4 = plot(ax, xx, 2*pi*(xx - aL0)*pi/180, 'b--', 'LineWidth',1);
   ylabel(ax,'c_l  ,  c_d');
```

```
    yyaxis(ax,'right');
    h3 = errorbar(ax, a, res.ld, res.dld, 'k^-', 'MarkerFaceColor','k', 'LineWidth',1.2);
    ylabel(ax,'c_l / c_d');
    ax.YAxis(1).Color = 'k';  ax.YAxis(2).Color = 'k';
    grid(ax,'on');  box(ax,'on');
    xlabel(ax,'\alpha (deg)');
    title(ax, [name ': lift, drag and lift-to-drag']);
    legend(ax, [h1 h2 h3 h4], {'c_l','c_d','c_l/c_d','c_l, thin airfoil theory'},
'Location','northwest');
    save_fig(fig, fullfile(outDir, ['Fig_Aero_' tag '.png']), DPI);
end
function plot_aero_panels(res, tag, name, aL0, fitRange, outDir, DPI)
% same data, three separate panels (cd is small so this is easier to read)
    a = res.alpha;
    fig = figure('Color','w','Position',[50 50 1450 430]);
    ax = subplot(1,3,1);  hold(ax,'on');
    errorbar(ax, a, res.cl, res.dcl, 'bo-', 'MarkerFaceColor','b', 'DisplayName','Measured');
    xx = linspace(min(a), max(a), 50);
    plot(ax, xx, 2*pi*(xx - aL0)*pi/180, 'k--', 'DisplayName','Thin airfoil theory');
    in = a >= fitRange(1) & a <= fitRange(2) & ~isnan(res.cl);
    p = polyfit(a(in), res.cl(in), 1);
    xf = linspace(fitRange(1), fitRange(2), 20);
    plot(ax, xf, polyval(p, xf), 'r-', 'LineWidth',1.5, 'DisplayName','Linear fit');
    legend(ax, 'Location','northwest');
    grid(ax,'on');  box(ax,'on');  xlabel(ax,'\alpha (deg)');  ylabel(ax,'c_l');  title(ax,'Lift
curve');
    ax = subplot(1,3,2);  hold(ax,'on');
    errorbar(ax, a, res.cd, res.dcd, 'rs-', 'MarkerFaceColor','r');
    grid(ax,'on');  box(ax,'on');  xlabel(ax,'\alpha (deg)');  ylabel(ax,'c_d');  title(ax,'Drag
curve');
    ax = subplot(1,3,3);  hold(ax,'on');
    errorbar(ax, a, res.ld, res.dld, 'k^-', 'MarkerFaceColor','k');
    grid(ax,'on');  box(ax,'on');  xlabel(ax,'\alpha (deg)');  ylabel(ax,'c_l/c_d');
title(ax,'Lift-to-drag ratio');
    sgtitle(name);
    save_fig(fig, fullfile(outDir, ['Fig_AeroPanels_' tag '.png']), DPI);
end
function save_fig(fig, fname, DPI)
    try, theme(fig,'light'); end        % forces white axes/black text (R2025a+)
try
        exportgraphics(fig, fname, 'Resolution', DPI, 'BackgroundColor','white');
catch
        print(fig, fname, '-dpng', sprintf('-r%d', DPI));
end
end
function r = aero_plot_data(res)
    % 0012 negative angles are mirror images, and angles where saturated taps
    % removed the suction peak are not valid, so leave both off the plots.
    k = res.alpha >= 0 & res.nDrop <= min(res.nDrop) + 1;
    r = struct('alpha',res.alpha(k), 'cl',res.cl(k), 'cd',res.cd(k), 'ld',res.ld(k), ...
               'dcl',res.dcl(k), 'dcd',res.dcd(k), 'dld',res.dld(k));
    r.ld(r.cd <= 0) = NaN;                        % l/d is meaningless when cd <= 0
    skipped = res.alpha(res.alpha >= 0 & ~k);
    if ~isempty(skipped)
        fprintf('Aero plots skip alpha = %s deg (saturated taps)\n', num2str(skipped));
    end
end
```

References

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